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Scattering by a Two-Dimensional Periodic Array of Conducting Plates

CHAO-CHUN CHEN

***Abstract*—The boundary value problem of an infinite array of thin plates arranged in a doubly periodic grid along any two coordinates is formulated in a general form for an arbitrarily polarized plane wave incident from any oblique angle. The induced current on the plate, the near-field distribution, and the distant reflected waves can be obtained to a very close accuracy. Both magnitudes and phases of the reflection coefficients for some specific examples are determined explicitly. For the case of a wave incident normally on a rectangular lattice array of narrow rectangular plates, the calculated values are in excellent agreement with the measurements in a previously published paper.**

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Introduction

An INFINITE array of metallic plates or strips such as is illustrated in Fig. 1, forms a useful model for the analysis of many practical microwave structures such as filters, lens, and artificial dielectrics [1]. A knowledge of the reflection and transmission coefficients at the array face is required in each of these applications.

The problem of scattering by a two-dimensional periodic array of rectangular plates was investigated by Ott, Kouyoumjian, and Peters [2]. They used the point matching method to solve the integral equation for the unknown current on each plate. The solution given is restricted to

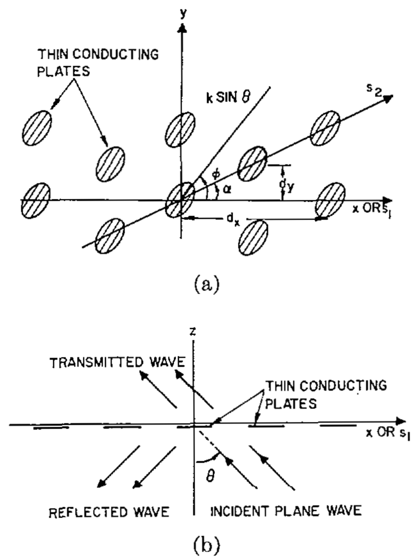

Fig. 1. Geometry of scattering of plane wave by periodic array of thin conducting plates. (a) Front view. (b) Side view.

the case of narrow plates arranged in a rectangular lattice with a normally incident plane wave. The complementary problem of scattering by a conducting screen perforated periodically with apertures was treated by Kieburtz and Ishimaru [3] by the variational method. The accuracy of the variational solution depends on the ability to choose an appropriate trial function.

A more general formulation of the scattering problem of a two-dimensional periodic array of plates is presented in this paper. The formulation applies to thin perfectly conducting plates of arbitrary shape distributed periodically along any two skewed coordinates with a plane wave of arbitrary polarization incident from any oblique angle. The size of the conducting plates may be of comparable width or less than the wavelength of the electromagnetic field. The procedure to be presented here is to expand the electromagnetic field distribution near the array of the conducting plates into a set of Floquet mode functions. By requiring the total electric field to vanish on the conducting plates, an integral equation for the unknown current on each plate is obtained. To solve this integral equation the unknown current is first expressed by a complete set of orthonormal mode functions and then its mode coefficients are determined by the method of moments [4]. The accuracy of the induced current on the plates and the reflection and transmission coefficients thus obtained are dependent upon the number of modes retained in the computation. A very accurate solution can be obtained by the high-speed computer. Numerical examples for rectangular plate arrays are given that are found to be in excellent agreement with the measurements made by Ott, Kouyoumjian, and Peters [2].

Theoretical Analysis

An infinite planar array is considered that consists of conducting plates arranged periodically along skewed coordinates s_1 and s_2 that enclose the angle α, as shown in Fig. 1. In the figure, θ is the angle between the propagation vector $\bar{k}$ and the normal to the plane of the array, and ϕ is the angle between the x axis and the projection of $\bar{k}$ on the x–y plane. All the plates in the array are assumed to be identical and infinitesimally thin. The electromagnetic fields must satisfy the periodicity requirements imposed by Floquet's theorem. Hence, the scalar wave equation has the following solution [5] with the exp $(j\omega t)$ time dependence omitted:

$$\psi_{pq} = \exp\left[-j(U_{pq}x + V_{pq}y + W_{pq}z)\right] \tag{1}$$

where

$$U_{pq} = k \sin\theta \cos\phi + 2\pi p/d_x \tag{2}$$

$$V_{pq} = k \sin\theta \sin\phi + 2\pi q/d_y - 2\pi p/(d_x \tan\alpha), \tag{3}$$

$$\text{for } p,q = 0, \pm1, \pm2, \cdots, \pm\infty$$

$$\begin{aligned} W_{pq} &= (k^2 - T_{pq}^2)^{1/2}, && \text{for } k^2 > T_{pq}^2 \\ &= -j\,|(k^2 - T_{pq}^2)^{1/2}|, && \text{for } k^2 < T_{pq}^2 \end{aligned} \tag{4}$$

with

$$T_{pq}^2 = U_{pq}^2 + V_{pq}^2$$

where p and q are the grating harmonics because of the periodicity present in the array. The modal propagation constant W_{pq} is positive for propagating modes and negative imaginary for evanescent modes in (1). The number of propagating modes in free space depends on the element spacings as well as the direction of the propagation vector $\bar{k}$. For the isosceles triangular array

$$(\lambda/d_x)^2 + (\lambda/2d_y)^2 \geq (1 + \sin\theta)^2$$

and

$$2(\lambda/d_x), (\lambda/d_y) \geq (1 + \sin\theta)$$

only the modes with p and q both equal to zero propagate; otherwise higher order modes may also exist in the distant scattered field.

The orthonormal mode functions for the transverse electric field components are expressible in terms of the scalar wave function ψ and the resulting TE and TM mode functions, transverse with respect to the z axis, are

$$\bar{\Phi}_{pq}^{\text{TE}} = \frac{1}{(d_x d_y)^{1/2}} \left(\frac{V_{pq}}{T_{pq}}\hat{x} - \frac{U_{pq}}{T_{pq}}\hat{y}\right)\psi_{pq}, \quad \text{for TE modes} \tag{5}$$

$$\bar{\Phi}_{pq}^{\text{TM}} = \frac{1}{(d_x d_y)^{1/2}} \left(\frac{U_{pq}}{T_{pq}}\hat{x} + \frac{V_{pq}}{T_{pq}}\hat{y}\right)\psi_{pq}, \quad \text{for TM modes.} \tag{6}$$

The transverse electric and magnetic fields are related by the modal impedances, such as

$$\eta_{pq}^{\text{TE}} = \frac{k}{W_{pq}}\left(\frac{\mu_0}{\epsilon_0}\right)^{1/2} \tag{7}$$

$$\eta_{pq}^{\text{TM}} = \frac{W_{pq}}{k}\left(\frac{\mu_0}{\epsilon_0}\right)^{1/2} \tag{8}$$

Since an electromagnetic plane wave can always be decomposed into a combination of E- and H-polarized plane waves that correspond to the TE and TM Floquet modes with $p = 0$, $q = 0$, a plane wave with unit electric field intensity incident in the ϕ plane and at an oblique angle θ with the array normal can, therefore, be expressed as

$$\bar{E}^i = \sum_{r=1}^{2} A_{00r}\bar{\Phi}_{00r} \tag{9}$$

$$\bar{H}^i = \sum_{r=1}^{2} \frac{A_{00r}}{\eta_{00r}} (\hat{z} \times \bar{\Phi}_{00r}) \tag{10}$$

where A_{00r} is the magnitude of incident field component, and the third subscript $r = 1$ or 2 is used to designate, respectively, the TE and TM Floquet modes.

The scattered field can generally be expressed in terms of the Floquet modes with reflection coefficients R_{pqr}

$$\bar{E}^s = \sum_p \sum_q \sum_{r=1}^{2} R_{pqr}\bar{\Phi}_{pqr} \tag{11}$$

$$\bar{H}^s = -\sum_p \sum_q \sum_{r=1}^{2} (R_{pqr}/\eta_{pqr}) (\hat{z} \times \bar{\Phi}_{pqr}). \tag{12}$$

Because of the orthonormality of the mode function $\bar{\Phi}_{pqr}$, the unknown reflection coefficient R_{pqr} can be obtained from (12)

$$R_{pqr} = \eta_{pqr} \iint_{\text{plate}} \hat{z} \times \bar{H}^s \cdot \bar{\Phi}_{pqr}^* \, da \tag{13}$$

where Φ_{pqr}^* is the complex conjugate of Φ_{pqr}. The boundary condition on the conducting plates requires that

$$\bar{E}^i + \bar{E}^s = 0 \quad \text{over each plate} \tag{14}$$

$$2\hat{z} \times (\bar{H}^i + \bar{H}^s) = \bar{K} \quad \text{over each plate.} \tag{15}$$

Substitution of (9), (11), and (13) into (14) yields the integral equation

$$\sum_{r=1}^{2} A_{00r}\bar{\Phi}_{00r} = -\sum_p \sum_q \sum_{r=1}^{2} \eta_{pqr}\bar{\Phi}_{pqr} \iint_{\text{plate}} \hat{z} \times \bar{H}^s \cdot \bar{\Phi}_{pqr}^* \, da. \tag{16}$$

To solve (16), the induced current $-z \times \bar{H}^s$ can be expressed in terms of another set of modal functions $\bar{\Psi}_{mnl}$ that is appropriate for the geometry of the plate under consideration and satisfies the plate boundary conditions.

$$-\hat{z} \times \bar{H}^s = \sum_m \sum_n \sum_{l=1}^{2} B_{mnl}\bar{\Psi}_{mnl}$$

$$\text{over each plate, for } m,n = 0,1,2,4,\cdots,\infty. \tag{17}$$

The fact that the functions Ψ_{mnl} are orthonormal over a single plate provides a faster convergence than the Floquet type mode expression in (12) and reduces much of the complicated computation in the process of evaluating the unknown coefficients of the induced current. Again, the third subscript $l = 1$, or 2 is used to stand for TE or TM modes. Both sides of (16) are multiplied by the complex conjugate of Ψ_{mnl}, and the resulting expression is integrated over the plate. The result is

$$\sum_{r=1}^{2} A_{00r}C_{00r}^{*MNL} = -\sum_p \sum_q \sum_{r=1}^{2} \eta_{pqr}C_{pqr}^{*MNL} \cdot \iint_{\text{plate}} \hat{z} \times \bar{H}^s \cdot \bar{\Phi}_{pqr}^* \, da \tag{18}$$

where

$$C_{pqr}^{MNL} = \iint_{\text{plate}} \bar{\Psi}_{MNL} \cdot \bar{\Phi}_{pqr}^* \, da. \tag{19}$$

The asterisks designate complex conjugates. The integral equation (18) can be employed to generate a system of linear algebraic equations with the mode coefficients B_{mnl} as unknowns. These algebraic equations can be written in a matrix form

$$[Z_{MNL}^{mnl}][B_{mnl}] = [D_{mnl}] \tag{20}$$

where $[Z_{MNL}^{mnl}]$ is a square impedance matrix in which the row index is designated by M,N,L and the column index is designated by m,n,l. The matrix elements are given by

$$[Z_{MNL}^{mnl}] = \sum_p \sum_q \sum_{r=1}^{2} \eta_{pqr}C_{pqr}^{*MNL}C_{pqr}^{mnl} \tag{21}$$

and

$$D_{mnl} = \sum_{r=1}^{2} A_{00r}C_{00r}^{*mnl}. \tag{22}$$

Equation (20) is a multiterminal network equation in which $[Z_{MNL}^{mnl}]$ is the impedance matrix. B_{mnl} is the branch current, D_{mnl} is the equivalent source voltage, and C_{pqr}^{mnl} is the coupling coefficient between two different types of modes.

A digital computer can be employed to calculate the reflection coefficient R_{pqr} and the induced current K on the plate from the relations given in (13) and (15). The unknown B_{mnl} of (20) is first obtained using the computer, and the values so obtained substituted into (17). To simplify the calculation of B_{mnl} and of the reflection coefficient, only the most significant TE and TM modes represented by (11) and (17) that satisfy the boundary conditions need to be chosen. For example, for an E-polarized plane wave incident in the x–z plane on the rectangular plates arranged symmetrically with the x axis, all TM Floquet modes in (11) and those for n even in (17) are negligible.

Scattering by Rectangular Plates

For the problem of scattering from rectangular thin obstacles arranged in either a rectangular or a triangular lattice array, the complete set of orthonormal mode functions Ψ_{mnl} in (17) for the induced current $-z \times \bar{H}^s$ are essentially the dual field functions of the transverse electric

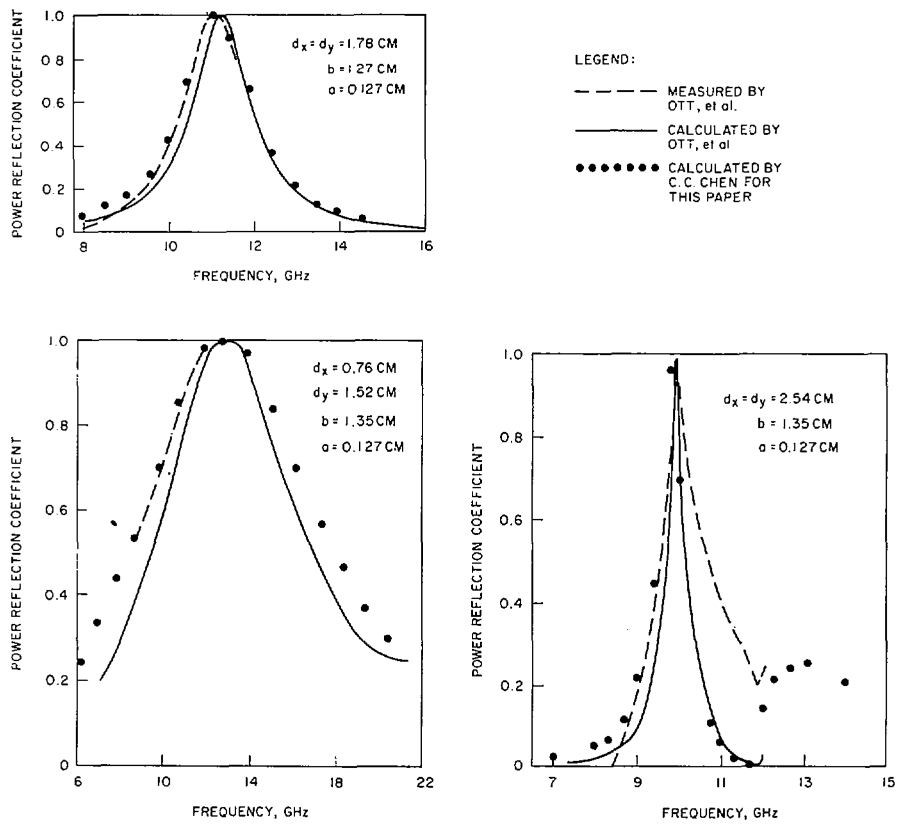

Fig. 2. Calculated and measured power reflection coefficients (after [1]) of narrow plates with rectangular lattice arrangement.

field functions for the rectangular waveguide [1]. They can be written as

$$\bar{\Psi}_{mn}^{\mathrm{TE}} = F[(m\pi/a)h_{mnx}\hat{x} + (n\pi/b)h_{mny}\hat{y}] \quad (23)$$

$$\bar{\Psi}_{mn}^{\mathrm{TM}} = F[(n\pi/b)h_{mny}\hat{x} - (m\pi/a)h_{mny}\hat{y}] \quad (24)$$

where

$$F = \left(\frac{\epsilon_m \epsilon_n}{ab}\right)^{1/2} \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{-1/2} \quad (25)$$

$$h_{mnx} = \sin\left(\frac{m\pi}{a}x - \frac{m\pi}{2}\right)\cos\left(\frac{n\pi}{b}y - \frac{n\pi}{2}\right) \quad (26)$$

$$h_{mny} = \cos\left(\frac{m\pi}{a}x - \frac{m\pi}{2}\right)\sin\left(\frac{n\pi}{b}y - \frac{n\pi}{2}\right) \quad (27)$$

and where a and b are the dimensions of the rectangular plate in the x and y directions and ϵ_m is the Newmann factor, ϵ_m equaling 1, for $m = 0$, and ϵ_m equaling 2, for $m \geq 1$. Substitution of (22) through (27) into (17) completes the formulation of the particular problem for the rectangular plate array.

Calculations using a digital computer were made for an arbitrarily polarized obliquely incident wave. In the computations, the lowest 10 modes for the rectangular plate and all Floquet modes whose transverse components of wave numbers T_{pq} were less than 10 times the wave number $k = 2\pi/\lambda$ were considered. In general, the number

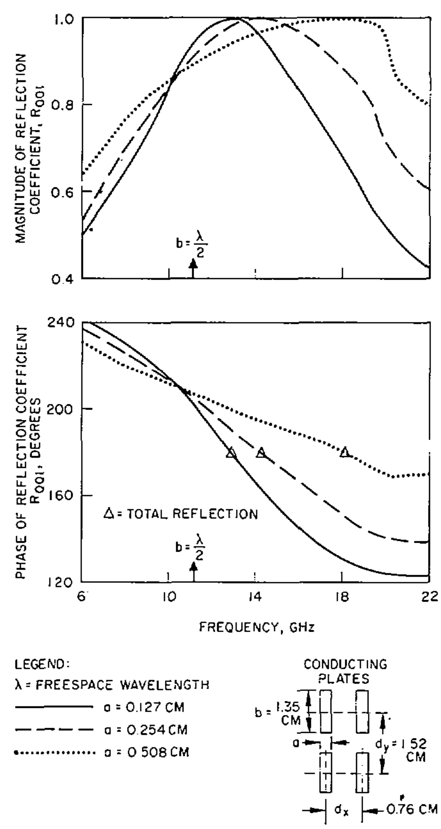

Fig. 3. Calculated reflection coefficients of rectangular lattice array of plates for plane wave with E field parallel to y axis incident normally to array.

TABLE I

THE SIX MOST SIGNIFICANT MODE COEFFICIENTS OF THE CURRENT INDUCED OVER A CONDUCTING PLATE BY A TE PLANE WAVE INCIDENT IN THE x–z PLANE

Mode Coefficient	$\theta = 1°$, $\phi = 0°$	$\theta = 31°$, $\phi = 0°$	$\theta = 61°$, $\phi = 0°$	$\theta = 81°$, $\phi = 0°$
B_{011}	$(7.49 - j0.204)$	$(6.39 - j0.473)$	$(3.59 - j0.319)$	$(1.16 - j0.0404)$
B_{031}	$(0.438 - j0.0119)$	$(0.342 - j0.0253)$	$(0.172 - j0.0152)$	$(0.0540 - j0.0018)$
B_{111}	$(0.0001 - j0.0045)$	$(-0.0090 - j0.122)$	$(-0.0117 - j0.131)$	$(-0.0017 - j0.0502)$
B_{131}	$(-0.0001 - j0.0061)$	$(-0.0114 - j0.153)$	$(-0.0132 - j0.149)$	$(-0.0019 - j0.0553)$
B_{112}	$(-0.0002 - j0.0098)$	$(-0.0244 - j0.330)$	$(-0.0392 - j0.440)$	$(-0.0062 - j0.1730)$
B_{132}	$(-0.0002 - j0.0100)$	$(-0.0183 - j0.248)$	$(-0.0209 - j0.235)$	$(-0.0030 - j0.0875)$

Note: All values are scaled to a factor of 10^{-3}. $d_x = 0.76$ cm, $d_y = 1.52$ cm, $\alpha = 90°$, $a = 0.127$ cm, $b = 1.35$ cm, $f = 13$ GHz.

of modes considered here is believed sufficient to obtain a solution for the reflection coefficient whose maximum error in magnitude is less than three percent of unity. For the case of a TE plane wave incident normally, the power reflection coefficients from three different arrays of narrow plates with the rectangular lattice arrangement ($\alpha = 90$ degrees) are illustrated in Fig. 2. The results are in excellent agreement with the values measured by Ott, Kouyoumjian, and Peters [2]. Since there is only one propagating mode in the far field, the total reflected wave is $R_{001}\bar{\Phi}_{001}$ which makes angle θ with the array normal on the opposite side of the incident wave.

The effect of twofold and fourfold increases in the initial value of the plate width are shown in Fig. 3. Total reflection occurs at the frequency at which the plate becomes resonant. The bandwidth increases with the plate width as expected; however, the resonance frequency also increases with plate width rather than decreasing.

The induced current on the conducting plate can be obtained from the relation of (15). Table I presents the coefficients B_{mnl} of a rectangular lattice array of thin conducting plates with $a = 0.127$ cm, $b = 1.35$ cm, $d_x = 0.76$ cm, and $d_y = 1.52$ cm, when the array is excited by a TE plane wave in the x–z plane. Due to the array symmetry the induced current for a normal angle of incidence is dominated by the $\bar{\Psi}_{011}$ and $\bar{\Psi}_{031}$ modes, which are the first and third sinusoidal terms along the y axis. As the angle of incidence θ increases, the coefficient of $\bar{\Psi}_{112}$ becomes larger, and then exceeds that of $\bar{\Psi}_{031}$. When a plane wave is incident in the planes, other than the x–z or y–z planes, due to the geometric dissymmetry of the array around the plane of incidence, there are many mode coefficients B_{mnl} that are of the same order of magnitude. For an accurate evaluation of the induced current when a plane wave is incident in these planes, additional modes must be considered in the computation. In general, the number of modes needed to describe the induced current distribution on each plate depends on both the angle of incidence and the plane of incidence.

The variations of the reflected TE and TM wave coefficients as a function of the angle of incidence when a TE plane wave is incident in different planes are shown in Fig. 4. It can be seen that the wave is totally reflected when the plane of incidence is the $\theta = 0$ degree plane and

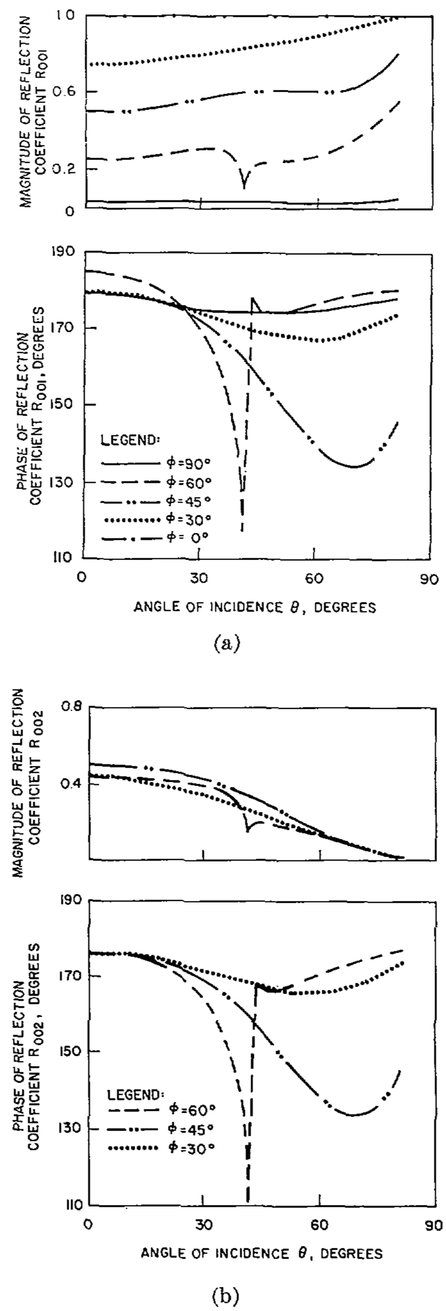

Fig. 4. Reflected TE and TM waves due to 13.0 GHz TE wave incident on rectangular lattice array (with dimensions as shown in Fig. 3). (a) TE waves. (b) TM waves.

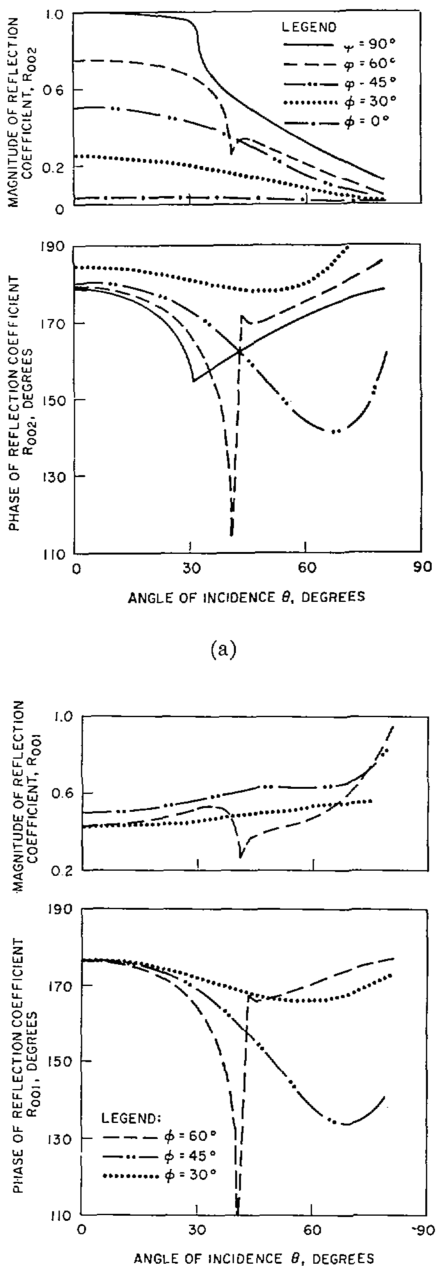

Fig. 5. Reflected TM and TE waves due to 13.0 GHz TM wave incident on rectangular lattice array (with dimensions as shown in Fig. 3). (a) TM waves. (b) TE waves.

almost totally transmitted when it is the $\theta = 90$ degrees plane. In addition, it may be seen that the reflected wave has no TM component when the plane of incidence is the x–z plane for which the array has geometric symmetry. The reflected TE wave component increases with angle of incidence θ, and the TM wave component decreases with the angle of incidence θ.

The variations of the reflected TM and TE wave coefficients as a function of the angle of incidence θ when a TM plane wave is incident in different planes are presented in Fig. 5.

The preceding computation has been made for the case of rectangular plates arranged in a rectangular lattice array. The element spacings of the array are so close that in the distant scattered field only the $p = 0$, $q = 0$ modes propagate. However, the formulation is general and applies equally well for arrays with large element spacings and/or arrays of circular disks. As the element spacing increases a point will be reached where the next higher order Floquet modes with either p or q not equal to zero can propagate. However, because of the mutual couplings among the elements, a total reflection may sometimes occur before the higher order modes become propagative. This phenomenon which is usually referred to as Wood's anomaly, is also predictable by the calculation presented here.

Conclusions

A generalized formulation of the scattering by a two-dimensional array of periodically arranged thin conducting plates was developed and used to calculate the reflection coefficients and induced current on a conducting plate. These solutions can be obtained to a high degree of accuracy within a few percent of the exact solutions. The accuracy depends upon the number of modes used to approximate the induced current on each plate and the number of Floquet modes used to approximate the near field of distribution.

The numerical results demonstrate that the solution for the rectangular plate configuration given in this paper possesses the following properties. 1) At resonance, the reflection coefficient is negative real and precisely equal to unity, while off resonance, its magnitude decays to less than unity. 2) At normal incidence, the relation between the reflected TM wave due to an incident TE plane wave, or vice versa, satisfies the reciprocity theorem. 3) The energy flow of the reflected and transmitted waves satisfies the law of energy conservation. 4) All numerical results presented in this paper agree by the application of Babinet's principle with the solution for a complementary problem of a conducting screen perforated with slots [6].

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