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DESIGN OF METAMATERIAL MULTILAYER STRUCTURES AS FREQUENCY SELECTIVE SURFACES

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Abstract—The reflection and transmission coefficients of multilayer structures are computed by the Transmission Line Transfer Matrix Method (TLTMM) and it is shown that metamaterials (MTMs) act as frequency selective surfaces (FSSs). Several examples of multilayer structures are analyzed, which are composed of combination of common materials and MTMs with dispersion relations. Interesting and uncommon behaviors are observed for MTMs. Novel applications are treated by TLTMM and a matrix method. The physical realizability of MTM structures and their optimum design are also described.

1. INTRODUCTION

Metallic frequency selective surfaces (FSSs) have been commonly fabricated by the printed circuit technology for the microwave and optical systems. Their frequency selective characteristics are determined by the shape and periodic spacing of conducting patches, which produce a periodic structure [1]. Metallic FSSs operate satisfactorily in the microwave frequency ranges, but their applications in the millimeter wave (MMW) and optical frequency ranges are accompanied by the increase in losses and thickness limitations of metal works.

Dielectric FSSs made up of common materials and structures have already been proposed in the literature and they have extensive applications in the microwave, millimetric wave (MMW) and optical frequency ranges. Various methods have been devised for the analysis of multilayer FSS made up of common dielectric materials, such as transmission line transfer matrix method (TLTMM) [2].

Common materials having positive permittivity ε and permeability μ are designated as double positive (DPS). MTMs are such materials that at least one of their permittivity or permeability constants is negative. MTMs are designated as epsilon-negative (ENG), mu-negative (MNG) and double negative (DNG) [3]. The entropy conditions show that simultaneously negative ε and μ are physically impossible in a nondispersive medium since they would violate the law of entropy [4].

In this paper, we study the macroscopic behavior of MTMs, which are considered as homogeneous and isotropic dielectrics having the real parts of ε or μ or bothnegative. Here it is shown that the multilayer MTM structures exhibit frequency selective properties, which are not observed in common dielectric materials (DPS).

Several numerical and analytical methods have already been devised for the analysis of multilayer MTM structures such as a matrix method [3], transmission line method [4], propagation matrix method [5], an iterative method [7], and developing the Green's functions in terms of plane waves [8].

It seems that there is a reciprocal relationship between FSS and MTM media. Namely, the MTM multilayered structures may exhibit properties of FSS and conversely FSS structures may be used as MTMs. In this paper, it is shown by several examples that multilayered MTM structures may behave as FSS. Analysis of wave propagation in multilayered structures composed of common dielectrics and MTMs is preformed by the extended Transmission Line Transfer Matrix Method (TLTMM) [2], and a matrix method [6]. Since the methods based on the equivalent transmission line model are simple, fast and accurate, so is TLTMM which facilitates the analysis, design and optimization of the problems. First, the problem formulation is briefly developed in MTM media and second several examples of the application of TLTMM are presented.

2. FORMULATION OF THE PROBLEM

In this paper, the analysis and design of MTMs as FSS have not been performed by any available commercial softwares, since bulk MTMs with negative ϵ_r and/or μ_r may not be defined in them. Moreover, design and optimization of antenna and microwave devices by available softwares are very CPU time consuming. Therefore, here, the design of MTM multilayer structures has been performed by TLTMM [2, 9] and a matrix method [6]. These two methods are based on the representation of forward and backward plane waves inside the layers of the structure.

2.1. TLTMM Method

TLTMM develops an equivalent transmission line for a multilayered planar structure under oblique incidence of an arbitrarily polarized plane wave. In this method, each transmission line section is defined by a characteristic impedance and propagation constant in the direction z normal to the structure, which depend on the angle of incidence, polarization and frequency. By TLTMM, the tangential components at the consecutive layers are related, which lead to two equations for two unknowns of reflection coefficient r and transmission coefficient t of the whole multilayer structure for TE, TM or elliptical polarizations. We may define reflectance ($R = rr^*$), transmittance ($T = tt^*$) and absorption ($A = 1 - R - T$) for the multilayer structure.

For the linear media, the reflectance due to the circularly polarized plane may be expressed in terms of those of TE and TM waves as

$$R^C = 0.5\left(R^{\mathrm{TE}} + R^{\mathrm{TM}}\right) \tag{1}$$

If the time dependence is defined as $\exp(j\omega t)$, the imaginary parts of ε and μ should be negative, to denote that losses satisfy the conservation of energy. Furthermore, the correct sings of square roots in the expressions for Z_ℓ and $\gamma_{\ell z}$ of the equivalent transmission lines of MTMs should be selected. That is, their signs should be selected in such a way that in lossy MTMs the real parts of Z_ℓ and $\gamma_{\ell z}$ should be positive and in lossless MTMs only Z_ℓ of MNG and $\gamma_{\ell z}$ of DNG should be negative. For details refer to [10].

2.2. Matrix Method

The potential functions ψ in the half free space and multilayered regions may be written as:

$$\begin{aligned}
\psi_0^{\mathrm{TE/TM}} &= e^{-\gamma_{0y}y}\left(e^{-\gamma_{0z}z} + r^{\mathrm{TE/TM}}e^{\gamma_{0z}z}\right)\\
\psi_\ell^{\mathrm{TE/TM}} &= e^{-\gamma_{\ell y}y}\left(C_\ell^{+}e^{-\gamma_{\ell z}z} + C_\ell^{-}e^{\gamma_{\ell z}z}\right), \quad \ell = 1, \ldots, N\\
\psi_{N+1}^{\mathrm{TE/TM}} &= e^{-\gamma_{\ell y}y}\left(t^{\mathrm{TE/TM}}e^{-\gamma^{z}_{\ell(z+1)}}\right)
\end{aligned} \tag{2}$$

where $r^{\mathrm{TE/TM}}$ and $t^{\mathrm{TE/TM}}$ are the reflection and transmission coefficients for TE or TM polarizations, respectively and $C_\ell^{\pm}$ are the normalized amplitudes of the forward and backward traveling waves in the ℓ'th layer, and $\vec{\gamma}_\ell = \hat{y}\gamma_{\ell y} + \hat{z}\gamma_{\ell z}$ is the propagation vector in the ℓ'th layer. There are $2N+2$ unknown amplitudes ($C_\ell^{\pm}$, $r^{\mathrm{TE/TM}}$ and $t^{\mathrm{TE/TM}}$)

which may be obtained from the continuity of tangential electric and magnetic fields at the $N+1$ boundaries by a matrix equation [6]

$$[A] \cdot [X] = [B] \tag{3}$$

3. EXAMPLES AND DISCUSSIONS

Several examples of oblique and normal incidence on different dispersive MTM multilayered media are treated to show new actual applications of MTMs as FSSs. All of the examples are designed by the least square error function, and the combination of conjugate gradient and genetic algorithm optimizer. The Drude and Lorentz dispersion models are used for ϵ_r and μ_r of the MTMs, respectively. The parameters of each model are obtained by appropriate optimization.

Assuming time dependence as $\exp(j\omega t)$, the permittivity of common lossy materials is assumed as in Eq. (4) which may be dispersive, the complex relative permittivities are assumed as in Eq. (5) according to the Drude's model and the complex relative permeabilities are assumed as in Eqs. (6) and (7) according to the Lorentz and resonance models, respectively, [11–13]

$$\varepsilon_c = \varepsilon' - j\left(\varepsilon'' + \sigma/\omega\right) \tag{4}$$

$$\varepsilon_r = 1 - f_{ep}^2 / \left(f^2 - j\Gamma_e f\right) \tag{5}$$

$$\mu_r = 1 - \left(f_{mp}^2 - f_{m0}^2\right) / \left(f^2 - f_{m0}^2 - j\Gamma_m f\right) \tag{6}$$

$$\mu_r = 1 - F f^2 / \left(f^2 - f_{m0}^2 - j\Gamma_m f\right) \tag{7}$$

where f_{ep} and f_{mp} are the electric and magnetic plasma frequencies, f_{m0} is the magnetic resonance frequency, Γ_e and Γ_m are the electric and magnetic damping factors, respectively and F is a relative factor between 0 and 1. The permittivities at microwave frequencies with the Drude's model may be realized by metallic thin wires [11] and permeabilities at microwave frequencies with the Lorentz and resonance models may be realized by a periodic array of resonators of various shapes such as rings, S, Ω, etc [12, 13]. Accordingly, ENG has $\mu_r = 1$ and ε_r in Eq. (5), MNG has $\varepsilon_r = 1$ and μ_r in Eqs. (6) or (7), and DNG has ε_r in Eq. (5) and μ_r in Eqs. (6) or (7) [13].

3.1. High Reflection Metamaterial Coating Including Dispersion

Design by MTMs reduces the number of layers and thickness of coating. For example the high reflection coating of 21 layers composed of

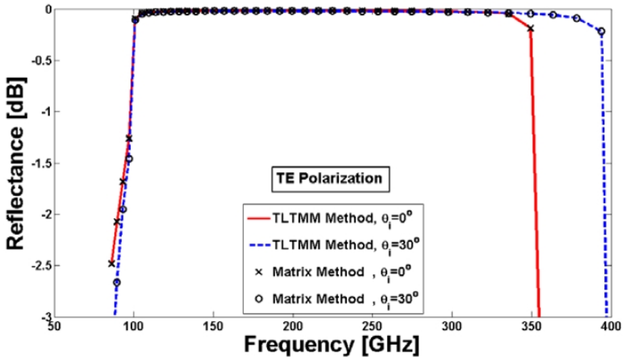

Figure 1. The transmittance of a high reflecting MNG MTM slab located in free space in the millimeter wave frequency band.

lossless and nondispersive DPS-ENG pairs has been analyzed in [7]. Whereas actual metamaterials are lossy and left-handed media are necessarily dispersive, in the present example the high reflection coating is designed only for one MNG layer having thickness 4.3 mm (which is equal to the thickness of 21 layers in [7]), $\varepsilon_r = 1$ and μ_r obeying dispersive relation (6) with constants $f_{mp} = 350\,\text{GHz}$, $f_{m0} = 100\,\text{GHz}$ and $\Gamma_m = 0.5\,\text{GHz}$. Fig. 1 shows the frequency dependence of transmittance of a high reflection coating for TE polarization under the incidence angles 0° and 30°. Design by common materials require large number of layers for high reflection coatings [14, 15].

3.2. Radar Absorbing Metamaterials

Common radar absorbing materials are made of dielectric, magnetic or frequency selective surface (FSS) layers. For example, the most widely used absorbing structures are the Dllenbach, Salisbury and Jaumann surfaces, which are narrow band resonant absorbers made up of lossy dielectric and magnetic layers on ground planes [16]. Lossy magnetic materials are occasionally used for common RAMs [17]. However, synthesis of optimum materials with nonunity permeability having the desired dispersion relations at microwave frequencies is difficult [18]. Therefore, MTMs with the desired effective permeabilities and dispersion relations are used in the proposed RAMs. The required permeabilities are obtained by split ring resonator (SRR), which makes MTMs and possess both the required magnetic losses and resonance behavior leading to RCS reduction.

Another type of common radar absorbing material may be made of a layer of FSS, which usually provides thin designs but is narrow band

and has acceptable performance only in a narrow range of incidence angles [19].

Radar absorbing metamaterials (RAMs) have various applications such as stealth and shielding of high reflection surfaces, and metal surfaces. Fabrication of wide band and wide angle RAMs being thin and having low weight will be interesting, which may be an improvement over the available technologies [20]. We design a wideband RAM under an oblique incidence $\theta_i = 30^\circ$ and TE, TM, and circularly polarized plane waves. The first layer is made of common lossy materials having a dispersion relation according to Eq. (4) and the second layer is composed of lossy MTMs with the dispersion relation in Eq. (6). The parameters of the layers are given in Table 1. The frequency response of a double layer DPS-MNG RAM under above conditions is shown in Fig. 2. It is observed that for frequencies from 1.2 to 18.6 GHz, the reflectance is less than 20 dB.

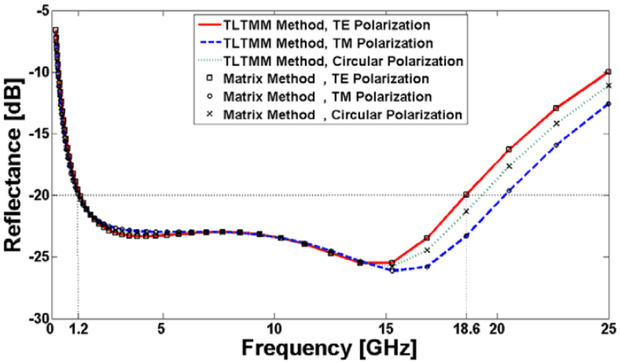

Figure 2. The reflectance of the DPS-MNG bilayer RAM coating composed of lossy common material and metamaterial under the plane wave incidence at incident angle of 30°.

3.3. Dichroic Plate

The dichroic plate reflects radio waves in a frequency band and transmits waves in another band. Presently, thick FSS sheets are used for dichroic plates, which have large dimensions and are heavy making them difficult to fabricate with concomitant mechanical problems [21]. Dichroic plates are used for feed systems of reflector antennas, which make it possible to use two independent feeds. Now, the usage of MTMs for the dichroic plates is a novel application, which has several advantages with respect to common materials, such as being lighter

Table 1. Specification of the DPS-MNG bilayer RAM where the first layer is composed of common lossy materials with $\mu_r = 1$ and the second layer is composed of split ring resonators with $\varepsilon_r = 1$.

Layer No.	1	2
Thickness (mm)	0.9	2.90
Dispersion relation	4	6
Dispersion Parameters	$\varepsilon_r = 1.7639$, $\sigma = 14.9751$	$f_{mp} = 22.2731\,\text{GHz}$, $f_{m0} = 2.4267\,\text{GHz}$, $\Gamma_m = 30\,\text{GHz}$

and less expensive, and having the required characteristics. A DNG dichroic plate is designed with ε_r in Eq. (5) and μ_r in Eq. (6), and in the *Ka* frequency band. The plate thickness is obtained as 4.5 mm, the parameters of the dispersion relations are obtained: $f_{ep} = 30.3679\,\text{GHz}$, $f_{mp} = 196.418\,\text{GHz}$, $f_{m0} = 90\,\text{GHz}$, $\Gamma_e = 11.930\,\text{GHz}$ and $\Gamma_m = 1\,\text{MHz}$ for the circularly polarized plane wave under the incident angle of 30°. The frequency responses of reflectance and transmittance of this dichroic plate are shown in Fig. 3. It is seen that in the frequency band 31.8–32.3 GHz, which is designed for reception, good reflection occurs, whereas in the band 34.2–34.7 GHz which is assigned for transmission, good transmission happens, and vice versa. These two frequency bands are used for the design of Mirror 7 of the deep space antenna of the European Space Agency installed in Spain [20].

3.4. Antenna Radome

Radomes are installed around radar and airplane antennas, which act as band pass filters. They pass operating frequencies with the least amount of insertion loss, whereas they reflect the out of band signal frequencies. Consequently, they tend to reduce the radar cross section (RCS) of antennas. Radomes should provide appropriate behavior for the incident radio waves in a specified frequency band and range of incident angles. Radomes are commonly made of dielectric FSS [22] or metallic FSS [23]. Dual band behavior of dielectric FSS are not reported which reduce RCS. Dual band FSS composed of embedded metallic pieces are thick and heavy. However, in this example we investigate the applicability of MTMs for the design and fabrication of radomes. Considered radome is a tri-layer MNG-MTM. Its relative permittivity is $\varepsilon_r = 1$ and its μ_r is assumed to obey the dispersion relation (6). The appropriate optimized parameters are obtained according to Table 2. The frequency response of its transmittance in the C band is shown in Fig. 4. The complete transmission about frequencies 4.5 and 5.5 GHz are seen in this figure.

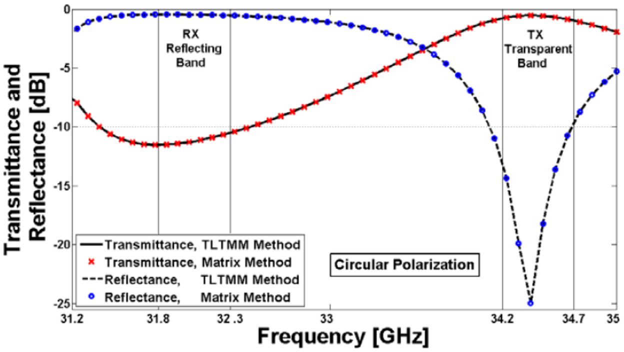

Figure 3. The transmittance and reflectance frequency responses of a dichroic plate in the Ka frequency band at incident angle of 30°.

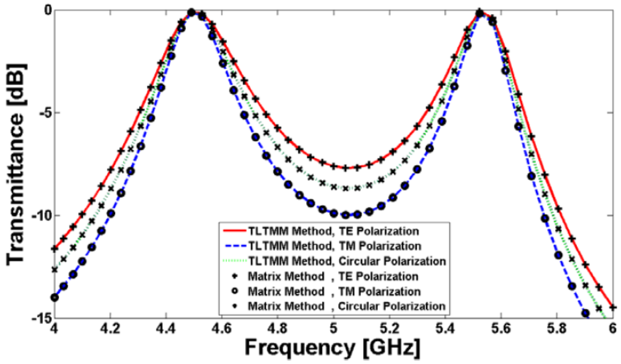

Figure 4. The transmittance-frequency dependence of a dual-band antenna radome in the C band at incident angle of 30°.

3.5. Dual Band Reflectors MTM

Dual band reflectors using FSS have many applications, such as Voyager and Cassini satellites [24, 25]. Today the applications of indoor wireless LAN and Wi-Fi frequency bands are expanding. Stealth wallpaper is usually used for indoor information protection.

Table 2. Specifications of a tri-layer radome which is composed of split ring resonator with $\varepsilon_r = 1$.

Layer No.	1	2	3
Thickness (mm)	5.5	0.7	1.1
f_{mp} (GHz)	125	125	125
f_{m0} (GHz)	16.1244	7.9227	10.6441
Γ_m (MHz)	1	1	1

Fabrication of stealth wallpaper with MTM is a new application for MTM. For example a dual band reflector with six-layers of MNG-MTM is designed for the obstruction of data transmission at 2.4 GHz and 3.4 GHz. Fig. 5 shows the transmittance-frequency characteristic and Table 3 shows its designed parameters.

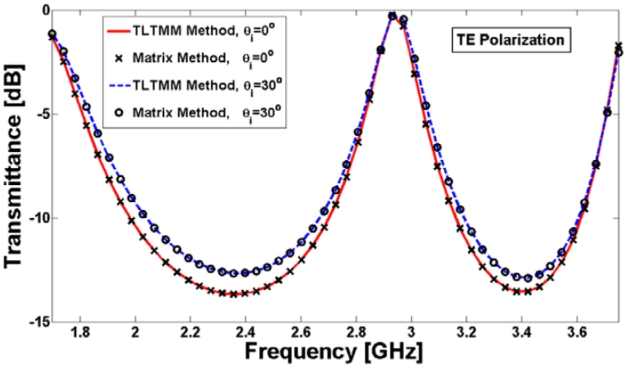

Figure 5. The transmittance-frequency dependence of a dual reflector in S band under TE polarization.

The proposed MTMs may be realized by thin wires (TWs), split ring resonators (SRRs) or some combination of them, under some constraints for their manufacturability. This can be down in two ways. By the first method, some constraints may be imposed on the parameters in the dispersion models, which are indirectly implemented on the geometrical dimensions of TWs and SRRs. Then, a system of nonlinear equations may be solved to determine the dimensions of TWs and SRRs from the parameters of dispersion models. By the second method, the dimensions of TWs and SRRs are directly obtained under constraints. The reflections among the parameters of Drude and resonance models and dimensions of TW and SRR elements are given in references [10, 11].

Table 3. Specifications of six-layers of MNG-MTM as a dual band reflector, composed of split ring resonators with $\varepsilon_r = 1$.

Layer No.	Thickness (mm)	f_{mp} (GHz)	f_{m0} (GHz)	$Gamma_m$ (MHz)
1	2.1	66.3169	8.9201	1
2	0.5	76.9876	4.3528	1
3	2.1	48.9533	4.3789	1
4	4.8	47.7790	5.9702	1
5	2	62.6087	57.1780	1
6	3.2	70.2682	31.0558	1

4. CONCLUSIONS

It is shown that the MTM multilayer structures act as FSS against the incident waves. The frequency responses of reflected and transmitted waves of such multilayer structures are accurately determined by a new transmission line transfer matrix method namely TLTMM and a matrix method, which demonstrate the dependence of their performance on the angle of incidence, polarization of the incident wave, physical parameters of the stratified media, and dispersion relations of materials.

Several new examples with novel applications were treated such as high reflection coatings, radar absorbing metamaterials, dichroic plates, antenna radomes and dual band reflector MTMs. The results of computations by TLTMM agree very well with other method.

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