Chapter 8

Plane Electromagnetic Waves

P.8-1 In a source-free simple medium,

Eq. (7-53b): $\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} = \sigma \bar{E} + \epsilon \frac{\partial \bar{E}}{\partial t}$. ①

Eq. (7-53a): $\nabla \times \nabla \times \bar{E} = \nabla \nabla \cdot \bar{E} - \nabla^2 \bar{E} = -\mu \frac{\partial}{\partial t}(\nabla \times \bar{H})$. ②

Substituting ① in ② and noting that $\nabla \cdot \bar{E} = 0$:

$$\nabla^2 \bar{E} - \mu\sigma \frac{\partial \bar{E}}{\partial t} - \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2} = 0.$$

Similarly for $\bar{H}$.

P.8-3 Assume that the vehicle moves with a velocity u in the $+z$ direction, which is the direction of propagation of the incident wave.

a) $\bar{E}_i = \bar{a}_x E_0 e^{j(\omega t - kz)}$, $\bar{E}_r = -\bar{a}_x E_0 e^{j(\omega' t - k' z)}$

$\bar{E}_i + \bar{E}_r = 0$ must be satisfied on reflecting surface for all t, $z = ut$:

$(\omega - ku)t = (\omega' + k'u)t$.

$\longrightarrow \omega' - \omega = -(k + k')u = -\left(\frac{\omega}{c} + \frac{\omega'}{c}\right)u = -(\omega + \omega')\frac{u}{c}$.

$\longrightarrow \frac{\omega'}{\omega} = 1 - \frac{u}{c}\left(1 + \frac{\omega'}{\omega}\right)$.

$\longrightarrow \frac{\omega'}{\omega} = \frac{f'}{f} = \frac{1 - u/c}{1 + u/c} \cong 1 - \frac{2u}{c}$, for $u \ll c$.

$\longrightarrow \Delta f = f' - f = -\frac{2u}{c} f$.

b) For $\Delta f = -2.33 \times 10^3$ (Hz) and $f = 10.5 \times 10^9$ (Hz): $u = 120$ (km/hr) $= 74.6$ (mi/hr).

P.8-4 Harmonic time dependence: $e^{j\omega t}$; $\frac{\partial}{\partial t} \to j\omega$.

Phasors: $\bar{E} = \bar{E}_0 e^{-j\bar{k}\cdot\bar{R}}$, $\bar{H} = \bar{H}_0 e^{-j\bar{k}\cdot\bar{R}}$; where $\bar{E}_0$ and $\bar{H}_0$ are constant vectors.

Now: $\nabla(e^{-j\bar{k}\cdot\bar{R}}) = e^{-j\bar{k}\cdot\bar{R}} \nabla(-j\bar{k}\cdot\bar{R}) = e^{-j\bar{k}\cdot\bar{R}}\left[-j\nabla(k_x x + k_y y + k_z z)\right]$

$= -j(\bar{a}_x k_x + \bar{a}_y k_y + \bar{a}_z k_z) e^{-j\bar{k}\cdot\bar{R}} = -j\bar{k} e^{-j\bar{k}\cdot\bar{R}}$

Maxwell's equations:

$\nabla \times \bar{E} = \nabla(e^{-j\bar{k}\cdot\bar{R}}) \times \bar{E}_0 = -j\omega\mu\bar{H} \longrightarrow \bar{k} \times \bar{E} = \omega\mu\bar{H}$;

$\nabla \times \bar{H} = \nabla(e^{-j\bar{k}\cdot\bar{R}}) \times \bar{H}_0 = j\omega\epsilon\bar{E} \longrightarrow \bar{k} \times \bar{H} = -\omega\epsilon\bar{E}$;

$\nabla \cdot \bar{E} = \nabla(e^{-j\bar{k}\cdot\bar{R}}) \cdot \bar{E}_0 = 0 \longrightarrow \bar{k} \cdot \bar{E} = 0$; $\nabla \cdot \bar{H} = \nabla(e^{-j\bar{k}\cdot\bar{R}}) \cdot \bar{H}_0 = 0 \longrightarrow \bar{k} \cdot \bar{H} = 0$.

P.8-5 $\bar{H} = \bar{a}_z 4\times10^{-6}\cos(10^7\pi t - k_0 y + \frac{\pi}{4})$ (A/m).

a) $k_0 = \omega\sqrt{\mu_0\epsilon_0} = \frac{10^7\pi}{3\times10^8} = \frac{\pi}{30} = 0.105$ (rad/m).

$\lambda = 2\pi/k_0 = 60$ (m).

At $t = 3\times10^{-3}$ (s), we require the argument of cosine in $\bar{H}$:

$$10^7\pi(3\times10^{-3}) - \frac{\pi}{30}y + \frac{\pi}{4} = \pm n\pi + \frac{\pi}{2}, \quad n = 0, 1, 2, \cdots$$

$$\longrightarrow y = \pm 30n - 7.5 \text{ (m)} = 22.5 \pm n\lambda/2 \text{ (m)}.$$

P.8-6 Phasor: $\bar{E} = \bar{a}_x 2e^{-jz/\sqrt{3}} + \bar{a}_y j e^{-jz/\sqrt{3}}$ (V/m).

a) $\omega = 10^8$ (rad/s) $\longrightarrow f = 10^8/2\pi = 1.59\times10^7$ (Hz),

$\beta = 1/\sqrt{3}$ (rad/m) $\longrightarrow \lambda = 2\pi/\beta = 2\sqrt{3}\pi$ (m).

b) $u = \frac{c}{\sqrt{\epsilon_r}} = \frac{\omega}{\beta} \longrightarrow \epsilon_r = \left(\frac{\beta c}{\omega}\right)^2 = 3.$

c) Left-hand elliptically polarized.

d) $\eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{120\pi}{\sqrt{\epsilon_r}} = \frac{120\pi}{\sqrt{3}}$ (Ω).

$\bar{H} = \frac{1}{\eta}\bar{a}_z\times\bar{E} = \frac{\sqrt{3}}{120\pi}(\bar{a}_y 2e^{-jz/\sqrt{3}} - \bar{a}_x j e^{-jz/\sqrt{3}})$,

$\bar{H}(z,t) = \frac{\sqrt{3}}{120\pi}\left[\bar{a}_x\sin(10^8 t - z/\sqrt{3}) + \bar{a}_y\cos(10^8 t - z/\sqrt{3})\right]$ (A/m).

P.8-7 Let $\alpha = \omega t - kz$. $\bar{E} = \bar{a}_x E_{10}\sin\alpha + \bar{a}_y E_{20}\sin(\alpha+\psi) = \bar{a}_x E_x + \bar{a}_y E_y$.

$\frac{E_x}{E_{10}} = \sin\alpha$, $\quad \frac{E_y}{E_{20}} = \sin(\alpha+\psi) = \sin\alpha\cos\psi + \cos\alpha\sin\psi$

$$= \frac{E_x}{E_{10}}\cos\psi + \sqrt{1-\left(\frac{E_x}{E_{10}}\right)^2}\sin\psi.$$

$$\left(\frac{E_y}{E_{20}} - \frac{E_x}{E_{10}}\cos\psi\right)^2 = \left(1 - \frac{E_x}{E_{10}}\right)\sin^2\psi.$$

$$\left(\frac{E_y}{E_{20}\sin\psi}\right)^2 + \left(\frac{E_x}{E_{10}\sin\psi}\right)^2 - 2\frac{E_x E_y}{E_{10}E_{20}}\frac{\cos\psi}{\sin^2\psi} = 1, \qquad (1)$$

which is the equation of an ellipse. In order to find the parameters of the polarization ellipse, rotate the coordinate axes x-y counterclockwise by an angle θ to x'-y'. Assume the equation of the ellipse in terms of the new coordinates to be

$$\left(\frac{E_x'}{a}\right)^2 + \left(\frac{E_y'}{b}\right)^2 = 1. \qquad (2)$$

where $E_x' = E_x \cos\theta + E_y \sin\theta$, ③

and $E_y' = -E_x \sin\theta + E_y \cos\theta$. ④

Substituting ③ and ④ in ② and rearranging:

$$E_x^2\left(\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}\right)+E_y^2\left(\frac{\sin^2\theta}{a^2}+\frac{\cos^2\theta}{b^2}\right)-2E_xE_y\sin\theta\cos\theta\left(\frac{1}{b^2}-\frac{1}{a^2}\right)=1. \quad ⑤$$

Comparing ① and ⑤, we obtain

$$\begin{cases}\dfrac{\cos^2\theta}{a^2}+\dfrac{\sin^2\theta}{b^2}=\dfrac{1}{E_{10}^2\sin^2\psi}, & ⑥\\[2mm] \dfrac{\sin^2\theta}{a^2}+\dfrac{\cos^2\theta}{b^2}=\dfrac{1}{E_{20}^2\sin^2\psi}, & ⑦\\[2mm] \sin\theta\cos\theta\left(\dfrac{1}{b^2}-\dfrac{1}{a^2}\right)=\dfrac{\cos\psi}{E_{10}E_{20}\sin^2\psi}. & ⑧\end{cases}$$

Eqs. ⑥, ⑦, and ⑧ can be solved for three unknowns:

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2E_{10}E_{20}\cos\psi}{E_{10}^2-E_{20}^2}\right),$$

$$a=\sqrt{\frac{2}{\frac{1}{E_{10}^2}(1+\sec 2\theta)+\frac{1}{E_{20}^2}(1-\sec 2\theta)}}\,\sin\psi,$$

$$b=\sqrt{\frac{2}{\frac{1}{E_{10}^2}(1-\sec 2\theta)+\frac{1}{E_{20}^2}(1+\sec 2\theta)}}\,\sin\psi.$$

In particular, if $E_{10}=E_{20}=E_0$: $\theta=45°$, $a=\sqrt{2}E_0\cos\frac{\psi}{2}$, $b=\sqrt{2}E_0\sin\frac{\psi}{2}$.

P.8-8 Let an elliptically polarized plane wave be represented by the phasor (with propagation factor e^{-jkz} omitted):

a) $$\bar{E}=\bar{a}_xE_1 \pm \bar{a}_yE_2e^{j\alpha},$$

where E_1, E_2, and α are arbitrary constants.

Right-hand circularly polarized wave: $\bar{E}_{rc}=E_{rc}(\bar{a}_x-\bar{a}_y j)$;

Left-hand circularly polarized wave: $\bar{E}_{lc}=E_{lc}(\bar{a}_x+\bar{a}_y j)$.

If $E_{rc}=\frac{1}{2}(E_1 \pm jE_2e^{j\alpha})$ and $E_{lc}=\frac{1}{2}(E_1 \mp jE_2e^{j\alpha})$,

then $$\bar{E}=\bar{E}_{rc}+\bar{E}_{lc}.$$

b) Right-hand circularly polarized wave:

$$\bar{E}_{rc}=E(\bar{a}_x-\bar{a}_y j)$$
$$=E(\bar{a}_x\tfrac{1}{2}-\bar{a}_y j2)+E(\bar{a}_x\tfrac{1}{2}+\bar{a}_y j)$$
$$=\bar{E}_{e+}+\bar{E}_{e-},$$

where $\bar{E}_{e+}$ and $\bar{E}_{e-}$ are right-hand and left-hand elliptically polarized waves respectively.

Similarly for a left-hand circularly polarized wave:

$$\bar{E}_{lc}=E(\bar{a}_x+\bar{a}_y j)=E(\bar{a}_x\tfrac{1}{2}+\bar{a}_y j2)+E(\bar{a}_x\tfrac{1}{2}-\bar{a}_y j)$$
$$=\bar{E}'_{e-}+\bar{E}'_{e+}.$$

P.8-9 For conducting media: $k_c = \beta - j\alpha$.

$$k_c^2 = \beta^2 - \alpha^2 - 2j\alpha\beta$$
$$= \omega^2\mu\epsilon_c = \omega^2\mu\epsilon\left(1 - j\frac{\sigma}{\omega\epsilon}\right).$$
$$\therefore \quad \beta^2 - \alpha^2 = \mathcal{R}_e(k_c^2) = \omega^2\mu\epsilon, \qquad ①$$
$$\beta^2 + \alpha^2 = |k_c^2| = \omega^2\mu\epsilon\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}. \qquad ②$$

From ① and ② we obtain

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]^{1/2}, \quad \beta = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{1/2}.$$

P.8-10 All three metals are good conductors, $\left(\frac{\sigma}{\omega\epsilon}\right)^2 \gg 1$.

$$\alpha = \sqrt{\pi f \mu \sigma}, \qquad \delta = \frac{1}{\alpha}, \qquad \eta_c = (1+j)\frac{\alpha}{\sigma}.$$

a) $f = 60$ (Hz)

	η_c (Ω)	α (Np/m)	α (dB/m)	δ (m)
Copper	$2.02(1+j)\times10^{-6}$	0.117×10^{3}	1.02×10^{3}	8.53×10^{-3}
Silver	$2.08(1+j)\times10^{-6}$	0.121×10^{3}	1.05×10^{3}	8.27×10^{-3}
Brass	$3.86(1+j)\times10^{-6}$	0.061×10^{3}	0.53×10^{3}	16.3×10^{-3}

b) $f = 1$ (MHz)

	η_c (Ω)	α (Np/m)	α (dB/m)	δ (m)
Copper	$2.61(1+j)\times10^{-4}$	1.51×10^{4}	1.31×10^{5}	6.61×10^{-5}
Silver	$2.57(1+j)\times10^{-4}$	1.58×10^{4}	1.36×10^{5}	6.32×10^{-5}
Brass	$4.98(1+j)\times10^{-4}$	0.79×10^{4}	0.69×10^{5}	12.6×10^{-5}

c) $f = 1$ (GHz)

	η_c (Ω)	α (Np/m)	α (dB/m)	δ (m)
Copper	$8.25(1+j)\times10^{-3}$	4.79×10^{5}	4.16×10^{6}	2.09×10^{-6}
Silver	$8.01(1+j)\times10^{-3}$	4.93×10^{5}	4.28×10^{6}	2.03×10^{-6}
Brass	$15.6(1+j)\times10^{-3}$	2.51×10^{5}	2.18×10^{6}	3.98×10^{-6}

P.8-11 $f = 3\times10^9\ (Hz)$, $\epsilon_r = 2.5$, $\tan\delta_c = \frac{\epsilon''}{\epsilon'} = 10^{-2}$.

a) Eq. (8-48): $\alpha = \frac{\omega\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}} = \frac{\omega}{2}\left(\frac{\epsilon''}{\epsilon'}\right)\frac{\sqrt{\epsilon_r}}{c} = 0.497\ (Np/m)$.

$$e^{-\alpha x} = \frac{1}{2} \longrightarrow x = \frac{1}{\alpha}\ln 2 = 1.395\ (m).$$

b) Eq. (8-50): $\eta_c = \frac{1}{\sqrt{\epsilon_r}}\sqrt{\frac{\mu_0}{\epsilon_0}}\left(1 + j\frac{\epsilon''}{2\epsilon'}\right) = 238(1 + j0.005) = 238\underline{/0.29°}\ (\Omega)$,

Eq. (8-49): $\beta = \omega\sqrt{\mu\epsilon}\left[1 + \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right] = 31.6\pi\ (rad/m)$.

$\lambda = \frac{2\pi}{\beta} = 0.063\ (m)$,

$u_p = \frac{\omega}{\beta} = 1.8973\times10^8\ (m/s)$.

$u_g = \frac{1}{\frac{d\beta}{d\omega}} = \frac{c}{\sqrt{\epsilon_r}}\left[1 + \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right] = 1.8975\times10^8\ (m/s)$.

c) At $x = 0$, $\bar{E} = \bar{a}_y e^{j\pi/2}$.

$\bar{H} = \frac{1}{\eta_c}\bar{a}_x\times\bar{E} = \bar{a}_z\, 0.210\, e^{j(\frac{\pi}{2} - 0.0016\pi)}$.

$\bar{H}(x,t) = \bar{a}_z\, 0.210\, e^{-0.497x}\sin(6\pi10^9 t - 31.6\pi x + 0.332\pi)\ (A/m)$.

P.8-12 $\sigma/\omega\epsilon = 4/10^{10}\pi\times80\times\left(\frac{1}{36}\times10^{-9}\right) = 0.18$.

a) $\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]^{1/2} = 84\ (Np/m)$,

$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{1/2} = 300\pi\ (rad/m)$,

$\eta_c = \sqrt{\frac{\mu}{\epsilon_c}} = \frac{120\pi}{\sqrt{\epsilon_r}\left[1+(\sigma/\omega\epsilon)^2\right]^{1/4}}e^{\frac{j}{2}\tan^{-1}(\sigma/\omega\epsilon)} = 41.8\, e^{j0.0283\pi}\ (\Omega)$,

$u_p = \omega/\beta = 33.3\times10^6\ (m/s)$, $\lambda = 2\pi/\beta = 0.67\ (cm)$, $\delta = \frac{1}{\alpha} = 1.19\ (cm)$.

b) $e^{-\alpha y} = \frac{1}{10}$, $y = \frac{1}{\alpha}\ln 10 = 2.74\ (cm)$.

c) $\bar{H}(y,t) = \bar{a}_x\, 0.1\, e^{-84\times0.5}\sin(10^{10}\pi t - 300\pi\times0.5 - \pi/3)$

$= \bar{a}_x\, 5.75\times10^{-20}\sin(10^{10}\pi t - \pi/3)\quad (A/m)$.

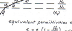
$\bar{E}(y,t) = \mathcal{I}_m\left[\eta_c\bar{H}(y)\times\bar{a}_y\right]e^{j\omega t} = \bar{a}_z\, 2.41\times10^{-18}\sin\left(10^{10}\pi t - \frac{\pi}{3} + 0.0283\pi\right)$ (V/m).

P.8-13 a) $\delta = \frac{1}{\sqrt{\pi f\mu\sigma}} \longrightarrow \sigma = \frac{1}{\pi f\mu\delta^2} = 0.99\times10^5\ (S/m)$.

b) At $f = 10^9\ (Hz)$, $\alpha = \sqrt{\pi f\mu\sigma} = 1.98\times10^4\ (Np/m)$.

$20\log_{10}e^{-\alpha z} = -30\ (dB) \longrightarrow z = \frac{1.5}{\alpha\log_{10}e} = 1.75\times10^{-4}\ (m) = 0.175\ (mm)$.

P.8-14



Assume the ionosphere to be stratified into layers having electron densities

$N_1 < N_2 < N_3 < \cdots < N_{max}$.

The corresponding equivalent permittivities of the layers are:

$$\epsilon_n = \epsilon_0\left(1 - \frac{f_{pn}^2}{f^2}\right) \quad \text{with} \quad f_{pn} = \frac{1}{2\pi}\sqrt{\frac{N_n e^2}{m\epsilon_0}}$$

and $\epsilon_i = \epsilon_0 > \epsilon_1 > \epsilon_2 > \epsilon_3 > \cdots > \epsilon_{min}$ (Corresponding to N_{max}).

From Snell's law of reflection:

$$\sin\theta_1 = \sin\theta_i\sqrt{\epsilon_0/\epsilon_1} = \sin\theta_i\Big/\sqrt{1-(f_{p1}/f)^2},$$

$$\sin\theta_2 = \sin\theta_1\sqrt{\epsilon_1/\epsilon_2} = \sin\theta_i\sqrt{\epsilon_0/\epsilon_2},$$

$$\sin\theta_3 = \sin\theta_2\sqrt{\epsilon_2/\epsilon_3} = \sin\theta_i\sqrt{\epsilon_0/\epsilon_3},$$

$$\vdots$$

For total reflection at the layer with ϵ_{min}, the angle of refraction $\theta_{max} = \pi/2$, and $\sin\theta_{max} = 1 = \sin\theta_i\sqrt{\epsilon_0/\epsilon_{min}}$.

$$\therefore \quad \epsilon_{min} = \epsilon_0\left(1 - f_{p,max}^2/f^2\right) = \epsilon_0\sin^2\theta_i.$$

$$\longrightarrow \quad f = f_{p,max}/\cos\theta_i = 9\sqrt{N_{max}}\,/\cos\theta_i.$$

P.8-15 a) From Eq. (8-72): $u_g = \dfrac{d\omega}{d\beta} = \dfrac{d}{d\beta}(\beta u_p) = u_p + \beta\dfrac{du_p}{d\beta}$.

b) $\lambda = \dfrac{2\pi}{\beta}$, $\dfrac{d\lambda}{d\beta} = -\dfrac{2\pi}{\beta^2} = -\dfrac{\lambda}{\beta}$.

$$u_g = u_p + \beta\left(\frac{du_p}{d\lambda}\frac{d\lambda}{d\beta}\right) = u_p - \lambda\frac{du_p}{d\lambda}.$$

P.8-16 $\mathcal{P}_{av} = |E|^2/2\eta_0 = 10^{-2}$ (W/cm²).

a) $|E| = \sqrt{0.02\eta_0} = 2.75$ (V/cm) $= 275$ (V/m).
$|H| = |E|/\eta_0 = 7.25\times10^{-3}$ (A/cm) $= 0.725$ (A/m).

b) $\mathcal{P}_{av} = |E|^2/2\eta_0 = 1300$ (W/m²).
$|E| = 990$ (V/m), $|H| = 2.63$ (A/m).

P.8-17 Assume circularly polarized plane wave:

$$\bar{E}(z,t) = \bar{a}_x E_0 \cos(\omega t - kz + \phi) + \bar{a}_y E_0 \sin(\omega t - kz + \phi).$$

$$\bar{H}(z,t) = \bar{a}_y \frac{E_0}{\eta} \cos(\omega t - kz + \phi) - \bar{a}_x \frac{E_0}{\eta} \sin(\omega t - kz + \phi).$$

Poynting vector, $\bar{\mathscr{P}} = \bar{E} \times \bar{H} = \bar{a}_z \frac{E_0^2}{\eta}\left[\cos^2(\omega t - kz + \phi) + \sin^2(\omega t - kz + \phi)\right]$

$$= \bar{a}_z \frac{E_0^2}{\eta}, \text{ a constant independent of } t \text{ and } z.$$

P.8-18 $\bar{E} = \bar{a}_\theta E_\theta + \bar{a}_\phi E_\phi$.

$$\bar{H} = \frac{1}{\eta}\bar{a}_R \times \bar{E} = \frac{1}{\eta}(\bar{a}_\phi E_\theta - \bar{a}_\theta E_\phi).$$

$$\bar{\mathscr{P}}_{av} = \frac{1}{2}\mathscr{R}e(\bar{E} \times \bar{H}^*) = \bar{a}_R \frac{1}{2\eta}(|E_\theta|^2 + |E_\phi|^2).$$

P.8-19 From Gauss's law, $\bar{E} = \bar{a}_r \frac{\rho}{2\pi\epsilon r}$, where ρ is the line charge density on the inner conductor.

$$V_0 = -\int_b^a \bar{E} \cdot d\bar{r} = \frac{\rho}{2\pi\epsilon}\ln\left(\frac{b}{a}\right) \longrightarrow \bar{E} = \bar{a}_r \frac{V_0}{r\ln(b/a)}.$$

From Ampère's circuital law, $\bar{H} = \bar{a}_\phi \frac{I}{2\pi r}$.

Poynting vector, $\bar{\mathscr{P}} = \bar{E} \times \bar{H} = \bar{a}_z \frac{V_0 I}{2\pi r^2 \ln(b/a)}$.

Power transmitted over cross-sectional area:

$$P = \int_S \bar{\mathscr{P}} \cdot d\bar{s} = \frac{V_0 I}{2\pi \ln(b/a)} \int_0^{2\pi}\int_a^b \left(\frac{1}{r^2}\right) r\,dr\,d\phi = V_0 I.$$

P.8-20 a) $\delta = \frac{1}{\sqrt{\pi f \mu_0 \sigma}}$, $f = 10^4/2\pi$.

b) $\bar{H}(x,t) = \bar{a}_y H_0 e^{-x/\delta}\cos\left(10^4 t - \frac{x}{\delta}\right)$.

$$\eta_c = (1+j)\frac{\alpha}{\sigma} = (1+j)\frac{1}{\sigma\delta} = \frac{\sqrt{2}}{\sigma\delta}e^{j\pi/4}.$$

$$E(x,t) = \bar{a}_x \frac{\sqrt{2}}{\sigma\delta} H_0 e^{-x/\delta}\cos\left(10^4 t - \frac{x}{\delta} + \frac{\pi}{4}\right).$$

c) $\bar{\mathscr{P}}_{av} = \frac{1}{2}\mathscr{R}e(\bar{E} \times \bar{H}^*) = \bar{a}_x \frac{1}{2}\frac{\sqrt{2}}{\sigma\delta}H_0^2 \cos\frac{\pi}{4}$

$$= \bar{a}_x \frac{1}{2}\left(\frac{H_0^2}{\sigma\delta}\right) \quad (\text{W/m}^2).$$

P.8-21 Given $\bar{E}_i = E_0(\bar{a}_x - j\bar{a}_y)e^{-j\beta z}$.

a) Assume reflected $\bar{E}_r(z) = (\bar{a}_x E_{rx} + \bar{a}_y E_{ry})e^{j\beta z}$.

Boundary condition at $z=0$: $\bar{E}_i(0) + \bar{E}_r(0) = 0$.

$\longrightarrow \bar{E}_r(z) = E_0(-\bar{a}_x + j\bar{a}_y)e^{j\beta z}$, a left-hand circularly polarized wave in $-z$ direction.

b) $\bar{a}_{n2} \times (\bar{H}_1 - \bar{H}_2) = \bar{J}_s \longrightarrow -\bar{a}_z \times [\bar{H}_i(0) + \bar{H}_r(0)] = \bar{J}_s$. ($\bar{H}_2 = 0$ in perfect conductor)

$\bar{H}_i(0) = \frac{1}{\eta_0}\bar{a}_z \times \bar{E}_i(0) = \frac{E_0}{\eta_0}(j\bar{a}_x + \bar{a}_y)$, $\quad \bar{H}_r(0) = \frac{1}{\eta_0}(-\bar{a}_z) \times \bar{E}_r(0) = \frac{E_0}{\eta_0}(j\bar{a}_x + \bar{a}_y)$.

$\bar{H}_1(0) = \bar{H}_i(0) + \bar{H}_r(0) = \frac{2E_0}{\eta_0}(j\bar{a}_x + \bar{a}_y)$,

$\bar{J}_s = -\bar{a}_z \times \bar{H}_1(0) = \frac{2E_0}{\eta_0}(\bar{a}_x - j\bar{a}_y)$.

c) $\bar{E}_1(z,t) = \mathcal{R}e\left[\bar{E}_i(z) + \bar{E}_r(z)\right]e^{j\omega t}$

$= \mathcal{R}e\, E_0\left[(\bar{a}_x - j\bar{a}_y)e^{-j\beta z} + (-\bar{a}_x + j\bar{a}_y)e^{j\beta z}\right]e^{j\omega t}$

$= \mathcal{R}e\, E_0\left[-2j(\bar{a}_x - j\bar{a}_y)\sin\beta z\right]e^{j\omega t}$

$= 2E_0 \sin\beta z\,(\bar{a}_x \sin\omega t - \bar{a}_y \cos\omega t)$.

P.8-22 Given $\bar{E}_i(x,z) = \bar{a}_y 10 e^{-j(6x+8z)}$ (V/m).

a) $k_x = 6$, $k_z = 8 \longrightarrow k = \beta = \sqrt{k_x^2 + k_z^2} = 10$ (rad/m).

$\lambda = 2\pi/k = 2\pi/10 = 0.628$ (m); $f = c/\lambda = 4.78\times10^8$ (Hz); $\omega = kc = 3\times10^9$ (rad/s)

b) $\bar{E}_i(x,z;t) = \bar{a}_y 10\cos(3\times10^9 t - 6x - 8z)$ (V/m).

$\bar{H}_i(x,z) = \frac{1}{\eta_0}\bar{a}_{ni} \times \bar{E}_i$ $\quad \left(\bar{a}_{ni} = \frac{\bar{k}}{k} = \bar{a}_x 0.6 + \bar{a}_z 0.8\right)$

$= \frac{1}{120\pi}(\bar{a}_x 0.6 + \bar{a}_z 0.8) \times \bar{a}_y 10 e^{-j(6x+8z)} = \left(-\bar{a}_x\frac{1}{15\pi} + \bar{a}_z\frac{1}{20\pi}\right)e^{-j(6x+8z)}$

$\bar{H}_i(x,z;t) = \left(-\bar{a}_x\frac{1}{15\pi} + \bar{a}_z\frac{1}{20\pi}\right)\cos(3\times10^9 t - 6x - 8z)$ (A/m).

c) $\cos\theta_i = \bar{a}_{ni}\cdot\bar{a}_z = \left(\frac{\bar{k}}{k}\right)\cdot\bar{a}_z = 0.8 \longrightarrow \theta_i = \cos^{-1}0.8 = 36.9°$.

d) $\bar{E}_i(x,0) + \bar{E}_r(x,0) = 0 \longrightarrow \bar{E}_r(x,z) = -\bar{a}_y 10 e^{-j(6x-8z)}$.

$\bar{H}_r(x,z) = \frac{1}{\eta_0}\bar{a}_{nr} \times \bar{E}_r(x,z)$ $\quad (\bar{a}_{nr} = \bar{a}_x 0.6 - \bar{a}_z 0.8)$

$= -\left(\bar{a}_x\frac{1}{15\pi} + \bar{a}_z\frac{1}{20\pi}\right)e^{-j(6x-8z)}$

e) $\bar{E}_1(x,z) = \bar{E}_i(x,z) + \bar{E}_r(x,z) = \bar{a}_y 10(e^{-j8z} - e^{j8z})e^{-j6x}$

$= -\bar{a}_y j20 e^{-j6x}\sin 8z$ (V/m).

$\bar{H}_1(x,z) = \bar{H}_i(x,z) + \bar{H}_r(x,z) = -\left(\bar{a}_x\frac{2}{15\pi}\cos 8z + \bar{a}_z\frac{j}{10\pi}\sin 8z\right)e^{-j6x}$ (A/m).

P.8-23 Given $\bar{E}_i(y,z)=5(\bar{a}_y+\bar{a}_z\sqrt{3})e^{j6(\sqrt{3}y-z)}$ (V/m).

a) $k_y=-6\sqrt{3}$, $k_z=6 \longrightarrow k=\sqrt{k_y^2+k_z^2}=12$ (rad/m).

$\lambda=2\pi/k=\pi/6=0.524$ (m); $f=c/\lambda=5.73\times10^8$ (Hz); $\omega=kc=3.60\times10^9$ (rad/s).

b) $\bar{E}_i(y,z;t)=5(\bar{a}_y+\bar{a}_z\sqrt{3})\cos(3.60\times10^9t+6\sqrt{3}y-6z)$ (V/m).

$\bar{H}_i(y,z)=\frac{1}{\eta_0}\bar{a}_{ni}\times\bar{E}_i=\frac{1}{120\pi}(-\bar{a}_y\frac{\sqrt{3}}{2}+\bar{a}_z\frac{1}{2})\times5(\bar{a}_y+\bar{a}_z\sqrt{3})e^{j6(\sqrt{3}y-z)}$

$=\bar{a}_x(-\frac{1}{12\pi})e^{j6(\sqrt{3}y-z)}$

$\bar{H}_i(y,z;t)=\bar{a}_x(-\frac{1}{12\pi})\cos(3.60\times10^9t+6\sqrt{3}y-6z)$ (A/m).

c) $\cos\theta_i=\bar{a}_{ni}\cdot\bar{a}_z=\frac{1}{2} \longrightarrow \theta_i=\cos^{-1}(\frac{1}{2})=60°$.

d) Conditions $\bar{a}_{nr}\cdot\bar{E}_r(y,z)=0$ and $E_{iy}(y,0)+E_{ry}(y,0)=0$ lead to:

$\bar{E}_r(y,z)=5(-\bar{a}_y+\bar{a}_z\sqrt{3})e^{j6(\sqrt{3}y+z)}$ (V/m),

$\bar{H}_r(y,z)=\frac{1}{\eta_0}\bar{a}_{nr}\times\bar{E}_r(y,z)=\frac{1}{120\pi}(-\bar{a}_y\frac{\sqrt{3}}{2}-\bar{a}_z\frac{1}{2})\times5(-\bar{a}_y+\bar{a}_z\sqrt{3})e^{j6(\sqrt{3}y+z)}$

$=\bar{a}_x(-\frac{1}{12\pi})e^{j6(\sqrt{3}y+z)}$ (A/m).

e) $\bar{E}_1(y,z)=\bar{E}_i(y,z)+\bar{E}_r(y,z)=(-\bar{a}_y j10\sin6z+\bar{a}_z10\sqrt{3}\cos6z)e^{j6\sqrt{3}y}$ (V/m),

$\bar{H}_1(y,z)=\bar{H}_i(y,z)+\bar{H}_r(y,z)=\bar{a}_x(-\frac{1}{6\pi})\cos6z\cdot e^{j6\sqrt{3}y}$ (A/m).

P.8-24 a) From Eqs. (8-113) and (8-114):

$\bar{E}_1(x,z;t)=\bar{a}_y2E_{i0}\sin(\beta_1z\cos\theta_i)\sin(\omega t-\beta_1x\sin\theta_i)$,

$\bar{H}_1(x,z;t)=\frac{2E_{i0}}{\eta_1}\left[-\bar{a}_x\cos\theta_i\cos(\beta_1z\cos\theta_i)\cos(\omega t-\beta_1x\sin\theta_i)\right.$
$\left.+\bar{a}_z\sin\theta_i\sin(\beta_1z\cos\theta_i)\sin(\omega t-\beta_1x\sin\theta_i)\right]$.

b) $\bar{\mathscr{P}}_{av}=\frac{1}{2}\mathcal{R}e(\bar{E}\times\bar{H}^*)=\bar{a}_x\frac{2E_{i0}^2}{\eta_1}\sin\theta_i\sin^2(\beta_1z\cos\theta_i)$.

P.8-25 a) From Eqs. (8-128) and (8-129):

$\bar{E}_1(x,z;t)=-2E_{i0}\left[\bar{a}_x\cos\theta_i\sin(\beta_1z\cos\theta_i)\cos(\omega t-\beta_1x\sin\theta_i)\right.$
$\left.+\bar{a}_z\sin\theta_i\cos(\beta_1z\cos\theta_i)\sin(\omega t-\beta_1x\sin\theta_i)\right]$,

$\bar{H}_1(x,z;t)=\bar{a}_y\frac{2E_{i0}}{\eta_1}\cos(\beta_1z\cos\theta_i)\sin(\omega t-\beta_1x\sin\theta_i)$.

b) $\bar{\mathscr{P}}_{av}=\frac{1}{2}\mathcal{R}e(\bar{E}\times\bar{H}^*)=\bar{a}_x\frac{2E_{i0}^2}{\eta_1}\sin\theta_i\cos^2(\beta_1z\cos\theta_i)$.

P.8-26 For normal incidence: $1+\Gamma=\tau$, where $|\Gamma|\le1$.

If $|\tau|=|\Gamma|$: $\Gamma<0$ and $\eta_1-\eta_2=2\eta_2 \longrightarrow \eta_1=3\eta_2 \longrightarrow |\Gamma|=\frac{1}{2}$.

$\therefore\ S=\frac{1+|\Gamma|}{1-|\Gamma|}=3 \ \text{———}\ S_{dB}=20\log_{10}3=9.54$ (dB).

P.8-27 a) In the lossy medium (medium 2):

$$\bar{E}_t = \bar{a}_x E_{t0} e^{-\alpha_2 z} e^{-j\beta_2 z},$$

where, from Problem P.8-9, $\alpha_2 = \omega\sqrt{\frac{\mu_2\epsilon_2}{2}}\left[\sqrt{1+\left(\frac{\sigma_2}{\omega\epsilon_2}\right)^2}-1\right]^{\frac{1}{2}}$, $\beta_2 = \omega\sqrt{\frac{\mu_2\epsilon_2}{2}}\left[\sqrt{1+\left(\frac{\sigma_2}{\omega\epsilon_2}\right)^2}+1\right]^{\frac{1}{2}}$.

Given: $\beta_1 = 6$ (rad/m) $\longrightarrow \omega = \beta_1 c = 1.8\times10^9$ (rad/s).

$\tan\delta_c = \frac{\sigma_2}{\omega\epsilon_2} = 0.5 \longrightarrow \alpha_2 = 2.30$ (Np/m), $\beta_2 = 9.76$ (rad/m)

$$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} = \frac{120\pi}{\sqrt{\epsilon_{r2}}(1+\tan^2\delta_c)^{1/4}} e^{j\frac{1}{2}\tan^{-1}(\sigma/\omega\epsilon)} = 225\, e^{j13.3°}.$$

$\bar{E}_t = \bar{a}_x E_{t0} e^{-2.30z} e^{-j9.76z}$, $\quad \bar{H}_t = \bar{a}_z \times \frac{\bar{E}_t}{\eta_2} = \bar{a}_y \frac{E_{t0}}{225} e^{-j13.3°} e^{-2.30z} e^{-j9.76z}$

Let $\bar{E}_r = \bar{a}_x E_{r0} e^{j6z} \longrightarrow \bar{H}_r = -\bar{a}_y \frac{E_{r0}}{120\pi} e^{j6z}$. $\quad \bar{H}_i = \bar{a}_y \frac{10}{120\pi} e^{-j6z}$.

Boundary conditions for $\bar{E}$ and $\bar{H}$ at $z=0$:

$$\begin{cases} 10 + E_{r0} = E_{t0}, \\ 10 - E_{r0} = E_{t0}\sqrt{\epsilon_{r2}}(1+\tan^2\delta_c)^{1/4} e^{-j13.3°}. \end{cases}$$

$\longrightarrow E_{r0} = 2.77\, e^{j157°}$; $\quad E_{t0} = 7.53\, e^{-j172°}$

$\therefore$

$\bar{E}_r(z,t) = \bar{a}_x 2.77 \cos(1.8\times10^9 t + 6z + 157°)$ (V/m),

$\bar{H}_r(z,t) = -\bar{a}_y 0.073 \cos(1.8\times10^9 t + 6z + 157°)$ (A/m),

$\bar{E}_t(z,t) = \bar{a}_x 7.53\, e^{-2.30z} \cos(1.8\times10^9 t - 9.76z - 172°)$ (V/m),

$\bar{H}_t(z,t) = \bar{a}_y 0.033\, e^{-2.30z} \cos(1.8\times10^9 t - 9.76z + 176.7°)$ (A/m).

b) $(\bar{\mathscr{P}}_{av})_1 = \bar{a}_z\left(\frac{10^2}{2\times120\pi} - \frac{2.77^2}{2\times120\pi}\right) = \bar{a}_z 0.122$. (W/m²),

$(\bar{\mathscr{P}}_{av})_2 = \bar{a}_z \frac{7.53^2}{2\times225}(\cos 13.3°)\, e^{-4.60z} = \bar{a}_z 0.122\, e^{-4.60z}$ (W/m²).

P.8-28 a) $\Gamma = \frac{E_r}{E_i} = \frac{\eta_c - \eta_0}{\eta_c + \eta_0}$; $\quad \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi$, $\eta_c = \sqrt{j\omega\mu/\sigma}$.

$|\eta_c| \ll \eta_0$.

b) $|\Gamma|^2 = \left|\frac{\eta_c - \eta_0}{\eta_c + \eta_0}\right|^2 = \left|\frac{1-\eta_c/\eta_0}{1+\eta_c/\eta_0}\right|^2 \cong \left|1 - 2\eta_c/\eta_0\right|^2$

$= (1 - 2\eta_c/\eta_0)(1 - 2\eta_c^*/\eta_0) \cong 1 - 4\mathcal{R}_e(\eta_c)/\eta_0$.

Fraction of power absorbed, $F = 1 - |\Gamma|^2 = \frac{4}{\eta_0}\mathcal{R}_e\sqrt{\frac{j\omega\mu}{\sigma}}$

$= \frac{4}{\eta_0}\sqrt{\frac{\omega\mu}{2\sigma}}$.

c) $\omega = 2\pi\times10^6$ (Hz). For iron: $\mu = 4{,}000\times(4\pi 10^{-7})$ (H/m), $\sigma = 10^7$ (S/m).

$F = 4.21\times10^{-4}$, or 0.0421%.

P.8-29 From Eqs. (8-156) through (8-161):

$$\bar{E}_1 = \bar{a}_x (E_{i0} e^{-j\beta_0 z} + E_{r0} e^{j\beta_0 z}), \qquad \bar{H}_1 = \bar{a}_y \frac{1}{\eta_0}(E_{i0} e^{-j\beta_0 z} - E_{r0} e^{j\beta_0 z}),$$

$$\bar{E}_2 = \bar{a}_x (E_2^+ e^{-j\beta_2 z} + E_2^- e^{j\beta_2 z}), \qquad \bar{H}_2 = \bar{a}_y \frac{1}{\eta_2}(E_2^+ e^{-j\beta_2 z} - E_2^- e^{j\beta_2 z}),$$

$$\bar{E}_t = \bar{a}_x E_{t0} e^{-j\beta_0 z}. \qquad \bar{H}_t = \bar{a}_y \frac{1}{\eta_0} e^{-j\beta_0 z}.$$

Boundary conditions $\begin{cases} \text{at } z=0: & \bar{E}_1(0) = \bar{E}_2(0), \quad \bar{H}_1(0) = \bar{H}_2(0). \\ \text{at } z=d: & \bar{E}_2(d) = \bar{E}_t(d), \quad \bar{H}_2(d) = \bar{H}_t(d). \end{cases}$

Four equations to solve four unknowns E_2^+, E_2^-, E_{r0}, and E_{t0} in terms of E_{i0}:

a) $$E_{r0} = -\frac{j(\eta_2^2 - \eta_0^2)\tan\beta_2 d}{\eta_0\eta_2 + j(\eta_0^2 + \eta_2^2)\tan\beta_2 d} E_{i0}, \qquad \text{where}$$

$$E_2^+ = \frac{\eta_2(\eta_0 + \eta_2)e^{j\beta_2 d}}{\eta_0\eta_2\cos\beta_2 d + j(\eta_0^2 + \eta_2^2)\sin\beta_2 d} E_{i0}, \qquad \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi,$$

$$E_2^- = \frac{\eta_2(\eta_2 - \eta_0)e^{-j\beta_2 d}}{\eta_0\eta_2\cos\beta_2 d + j(\eta_0^2 + \eta_2^2)\sin\beta_2 d} E_{i0}, \qquad \eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}},$$

$$E_{t0} = \frac{2\eta_0\eta_2 e^{j\beta_0 d}}{\eta_0\eta_2\cos\beta_2 d + j(\eta_0^2 + \eta_2^2)\sin\beta_2 d} E_{i0}, \qquad \beta_0 = \frac{\omega}{c}, \quad \beta_2 = \omega\sqrt{\mu_2\epsilon_2}.$$

b) If $d = \lambda_2/4$, $\beta_2 d = \pi/2$. $E_{r0} = -\frac{\eta_2^2 - \eta_0^2}{\eta_0^2 + \eta_2^2} E_{i0}$.

$\therefore \Gamma = -\frac{\eta_2^2 - \eta_0^2}{\eta_0^2 + \eta_2^2} \neq 0$ unless $\eta_2 = \eta_0$ (a trivial case).

If $d = n\lambda_2/2$ $(n = 1, 2, 3, \cdots)$, $\tan\beta_2 d = 0 \longrightarrow \Gamma = 0$.

P.8-30 From Example 8-12: $\eta_2 = \sqrt{\eta_1\eta_3} \longrightarrow \epsilon_{2r} = \sqrt{\epsilon_{r1}\epsilon_{r3}} = 2$.

a) Wavelength of red light in dielectric coating: $\lambda_2 = \frac{0.75}{\sqrt{\epsilon_{r2}}} = 0.530\,(\mu m) \longrightarrow d = \frac{\lambda_2}{4} = 0.133\,(\mu m)$

b) For violet light: $\lambda_2' = \frac{0.42}{\sqrt{2}} = 0.297\,(\mu m)$.

$\frac{d}{\lambda_2'} = 0.447 \longrightarrow \beta_2 d = 0.894\pi$.

From Eq. (8-173) and using impedances normalized with respect to $\eta_1 = \eta_0$: $Z_2(0) = \eta_2 \frac{\eta_3 + j\eta_2\tan\beta_2 d}{\eta_2 + j\eta_3\tan\beta_2 d} = \frac{1}{\sqrt{2}} \frac{\frac{1}{2} + j\frac{1}{\sqrt{2}}\tan\beta_2 d}{\frac{1}{\sqrt{2}} + j\frac{1}{2}\tan\beta_2 d} = \frac{0.5 - j0.247}{1.0 - j0.247}$.

$\Gamma = \frac{Z_2(0) - 1}{Z_2(0) + 1} = 0.317 e^{j195°}$.

Percentage of power reflected $= |\Gamma|^2 \times 100\%$
$= (0.317)^2 \times 100\% = 10\%$.

P.8-31

$$\Gamma_0 = \frac{Z_2(0)-\eta_1}{Z_2(0)+\eta_1}, \quad Z_2(0) = \eta_2 \frac{\eta_3 + j\eta_2 \tan\beta_2 d}{\eta_2 + j\eta_3 \tan\beta_2 d}.$$

$$\Gamma_{12} = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}. \longrightarrow \frac{\eta_1}{\eta_2} = \frac{1-\Gamma_{12}}{1+\Gamma_{12}}.$$

$$\Gamma_{23} = \frac{\eta_3-\eta_2}{\eta_3+\eta_2}. \longrightarrow \frac{\eta_2}{\eta_3} = \frac{1-\Gamma_{23}}{1+\Gamma_{23}}.$$

$$\therefore \Gamma_0 = \frac{1+j\frac{\eta_2}{\eta_3}\tan\beta_2 d - \frac{\eta_1}{\eta_2}\left(\frac{\eta_2}{\eta_3}+j\tan\beta_2 d\right)}{1+j\frac{\eta_2}{\eta_3}\tan\beta_2 d + \frac{\eta_1}{\eta_2}\left(\frac{\eta_2}{\eta_3}+j\tan\beta_2 d\right)}$$

$$= \frac{1+j\frac{1-\Gamma_{23}}{1+\Gamma_{23}}\tan\beta_2 d - \frac{1-\Gamma_{12}}{1+\Gamma_{12}}\left(\frac{1-\Gamma_{23}}{1+\Gamma_{23}}+j\tan\beta_2 d\right)}{1+j\frac{1-\Gamma_{23}}{1+\Gamma_{23}}\tan\beta_2 d + \frac{1-\Gamma_{12}}{1+\Gamma_{12}}\left(\frac{1-\Gamma_{23}}{1+\Gamma_{23}}+j\tan\beta_2 d\right)}$$

$$= \frac{(\Gamma_{12}+\Gamma_{23}) + j(\Gamma_{12}-\Gamma_{23})\tan\beta_2 d}{(1+\Gamma_{12}\Gamma_{23}) + j(1-\Gamma_{12}\Gamma_{23})\tan\beta_2 d}.$$

P.8-32

$$\bar{E}_1 = \bar{a}_x\left(E_{i0}e^{-j\beta_1 z} + E_{r0}e^{j\beta_1 z}\right),$$

$$\bar{H}_1 = \bar{a}_y\frac{1}{\eta_1}\left(E_{i0}e^{-j\beta_1 z} - E_{r0}e^{j\beta_1 z}\right).$$

$$\bar{E}_2 = \bar{a}_x\left(E_2^+e^{-j\beta_2 z} + E_2^-e^{j\beta_2 z}\right).$$

$$\bar{H}_2 = \bar{a}_y\frac{1}{\eta_2}\left(E_2^+e^{-j\beta_2 z} - E_2^-e^{j\beta_2 z}\right).$$

At $z=d$, $\bar{E}_2 = 0 \longrightarrow E_2^- = -E_2^+e^{-j2\beta_2 d}$.

$$\bar{E}_2 = \bar{a}_x E_2^+\left[e^{-j\beta_2 z} - e^{j\beta_2(z-2d)}\right].$$

$$\bar{H}_2 = \bar{a}_y\frac{E_2^+}{\eta_2}\left[e^{-j\beta_2 z} + e^{j\beta_2(z-2d)}\right].$$

Boundary conditions at $z=0$: $E_1(0)=E_2(0) \longrightarrow E_{i0}+E_{r0} = E_2^+(1-e^{-j2\beta_2 d})$,
$H_1(0)=H_2(0) \longrightarrow E_{i0}-E_{r0} = E_2^+\frac{\eta_1}{\eta_2}(1+e^{-j2\beta_2 d})$.

$$E_2^+ = \frac{2\eta_2 E_{i0}}{(\eta_2+\eta_1)+(\eta_2-\eta_1)e^{-j2\beta_2 d}}.$$

$$E_{r0} = -\left(\frac{\eta_2 - j\eta_1\tan\beta_2 d}{\eta_2 + j\eta_1\tan\beta_2 d}\right)E_{i0}.$$

a) $\bar{E}_r(z,t) = \bar{a}_x E_{i0} \cos[\omega(t - \frac{z}{u_p}) + \theta]$, $\theta = \pi - 2\tan^{-1}\left(\frac{\eta_2}{\eta_0}\tan\beta_2 d\right)$.

b) $\bar{E}_1(z,t) = \bar{a}_x E_{i0} \left\{\cos\omega(t - \frac{z}{u_p}) + \cos[\omega(t - \frac{z}{u_p}) + \theta]\right\}$.

c) $$\bar{E}_2(z,t) = \bar{a}_x \frac{2\eta_2 E_{i0}}{\sqrt{2[(\eta_0^2+\eta_2^2)+(\eta_0^2-\eta_2^2)\cos 2\beta_2 d]}} \left\{\cos[\omega(t - \frac{z}{u_{p2}}) + \psi] - \cos[\omega(t + \frac{z}{u_{p2}}) - \frac{2\omega d}{u_{p2}} + \psi]\right\},$$

$$\psi = \tan^{-1}\left[\frac{(\eta_0-\eta_2)\sin 2\beta_2 d}{(\eta_0+\eta_2)+(\eta_0-\eta_2)\cos 2\beta_2 d}\right].$$

d) $(\bar{\mathcal{P}}_{av})_1 = \frac{1}{2}\mathcal{R}e(\bar{E}_1 \times \bar{H}_1^*) = 0$.

e) $(\bar{\mathcal{P}}_{av})_2 = 0$.

f) Let $E_r = -E_{i0} \longrightarrow \tan\beta_2 d = 0 \longrightarrow d = n\lambda_2/2$, $n = 0, 1, 2, \cdots$

P.8-33 $k_2 = \beta_2 - j\alpha_2 = (1-j)\frac{1}{\delta}$, $\alpha_2 = \beta_2 = \frac{1}{\delta} = \sqrt{\pi f \mu_2 \sigma_2}$.

$\eta_2 = (1+j)\frac{\alpha_2}{\sigma_2} << \eta_0$ at 10 (MHz).

a) From Problem P.8-29,

$$E_2^+ = \eta_2 H_2^+ \cong -j\left(\frac{\eta_2}{\eta_0}\right)\frac{e^{\alpha_2 d}e^{j\beta_2 d}E_{i0}}{\sin(\beta_2 - j\alpha_2)d}.$$

b) $$E_2^- = -\eta_2 H_2^- \cong -j\left(\frac{\eta_2}{\eta_0}\right)\frac{e^{-\alpha_2 d}e^{-j\beta_2 d}E_{i0}}{\sin(\beta_2 - j\alpha_2)d}.$$

c) $$E_{30} = E_{t0} = \eta_0 H_{30} \cong -j\left(\frac{\eta_2}{\eta_0}\right)\frac{2e^{j\beta_0 d}E_{i0}}{\sin(\beta_2 - j\alpha_2)d}.$$

d) $$E_{r0} \cong -\frac{E_{i0}}{1 - j\frac{\eta_2}{\eta_0}\cot(\beta_2 - j\alpha_2)d}$$

$$= -\left(1 + j\frac{\eta_2}{\eta_0}\frac{1 + j\tan\beta_2 d\tanh\alpha_2 d}{\tan\beta_2 d - j\tanh\alpha_2 d}\right)E_{i0}.$$

$$(\bar{\mathcal{P}}_{av})_1 = \frac{1}{2}\mathcal{R}e[(\bar{E}_{i0} \times \bar{H}_{i0}^*) - (\bar{E}_{r0} \times \bar{H}_{r0}^*)]$$

$$= \bar{a}_z \frac{\alpha}{\eta_0^2 \sigma}(A + B),$$

where $$\frac{1 + j\tan\beta_2 d\tanh\alpha_2 d}{\tan\beta_2 d - j\tanh\alpha_2 d} = A + jB.$$

$$(\mathcal{P}_{av})_1 = \frac{\alpha}{\eta_0^2\sigma}\frac{\sin\beta_2 d\cos\beta_2 d + \sinh\alpha_2 d\cosh\alpha_2 d}{(\sin\beta_2 d\cosh\alpha_2 d)^2 + (\cos\beta_2 d\sinh\alpha_2 d)^2}.$$

$$(\mathcal{P}_{av})_3 = \frac{1}{2\eta_0}|E_{30}|^2 = \frac{1}{\eta_0^3}\left(\frac{\alpha}{\sigma}\right)^2\frac{4E_{i0}^2}{(\sin\beta_2 d\cosh\alpha_2 d)^2 + (\cos\beta_2 d\sinh\alpha_2 d)^2}.$$

$$\therefore \quad \frac{(\mathcal{P}_{av})_2}{(\mathcal{P}_{av})_1} = \frac{4}{\eta_0}\left(\frac{\alpha}{\sigma}\right)\frac{1}{\sin\beta_c d\cos\beta_c d + \sinh\alpha_c d\cosh\alpha_c d}.$$

$$(\mathcal{P}_{av})_i = \frac{1}{2\eta_0}E_{io}^2.$$

$$\frac{(\mathcal{P}_{av})_3}{(\mathcal{P}_{av})_i} = \frac{8}{\eta_0^2}\left(\frac{\alpha}{\sigma}\right)^2\frac{1}{(\sin\beta_c d\cosh\alpha_c d)^2 + (\cos\beta_c d\sinh\alpha_c d)^2}.$$

At $f = 10^9$ (Hz), $\sigma = 5.80\times10^7$ (S/m), $\alpha_c = \beta_c = 4.785\times10^5$, $d = \delta = \frac{1}{\alpha_c}$.

$$\frac{(\mathcal{P}_{av})_3}{(\mathcal{P}_{av})_i} = 1.339\times10^{-11}.$$

P.8-34 Given $f = f_p/2$ and $\theta_i = 60°$.

$$\longrightarrow \eta_p = \eta_0/\sqrt{1-(f_p/f)^2} = -j\eta_0/\sqrt{3}, \quad \eta_2/\eta_0 = -j/\sqrt{3}.$$

From Eq. (8-185): $\sin\theta_t = \frac{\eta_2}{\eta_0}\sin\theta_i = -j/2$, $\cos\theta_t = \sqrt{5}/2$, $\cos\theta_i = 1/2$.

a) From Eq. (8-206): $\Gamma_\perp = \frac{(\eta_p/\eta_0)\cos\theta_i - \cos\theta_t}{(\eta_p/\eta_0)\cos\theta_i + \cos\theta_t} = e^{j209°}$.

From Eq. (8-207): $\tau_\perp = \frac{2(\eta_p/\eta_0)\cos\theta_i}{(\eta_p/\eta_0)\cos\theta_i + \cos\theta_t} = 0.5\,e^{-j75.5°}$.

b) From Eq. (8-221): $\Gamma_\parallel = \frac{(\eta_p/\eta_0)\cos\theta_t - \cos\theta_i}{(\eta_p/\eta_0)\cos\theta_t + \cos\theta_i} = e^{j76°}$.

From Eq. (8-222): $\tau_\parallel = \frac{2(\eta_p/\eta_0)\cos\theta_i}{(\eta_p/\eta_0)\cos\theta_t + \cos\theta_i} = 0.177\,e^{-j38°}$.

$|\Gamma_\perp| = |\Gamma_\parallel| = 1$, but the phase shift of the reflected wave depends on the polarization of the incident wave. There are standing waves in the air and exponentially decaying transmitted waves in the ionosphere.

P.8-35 $k_{2x}^2 + k_{2z}^2 = k_2^2 = \omega^2\mu_0\epsilon_2 - j\omega\mu_0\sigma_2$. ①

Continuity conditions at $z=0$ for all x and y require:

$$k_{2x} = k_{1x} = \omega\sqrt{\mu_0\epsilon_0}\sin\theta_i = \beta_x = 2.09\times10^{-4}. \quad ②$$

$$k_{2z} = \beta_{2z} - j\alpha_{2z}. \quad ③$$

Combining ①, ② and ③, we can solve for α_{2z} and β_{2z} in terms of ω, μ_0, ϵ_2, σ_2, and β_x. But, since

$$\beta_x^2 << \omega^2\mu_0\epsilon_2 .$$

we have $\alpha_2 = \alpha_{2x} = \beta_{2x} \cong \frac{1}{\delta} = \sqrt{\pi f \mu_0 \sigma_2} = 0.3974 \ (m^{-1})$.

a) $\theta_t = \tan^{-1}\frac{\beta_x}{\beta_{2x}} = \tan^{-1}\frac{2.09}{0.3974}\times 10^{-6} \cong 5.26\times 10^{-6} \ (rad)$
$= 0.03°$.

b) $\Gamma_\parallel = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_0\cos\theta_i}$ $\qquad \eta_2 = \frac{\alpha_2}{\sigma_2}(1+j) = 0.0993(1+j)$.

$= \frac{2\times 0.0993(1+j)}{0.0993(1+j) + 377\cos 35°}$ $\qquad \cos\theta_t = \cos 0.03° \cong 1$.

$\cong 0.0151(1+j) = 0.0214\, e^{j\pi/4}$.

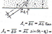
c) $(\mathcal{P}_{av})_i = \frac{E_{i0}^2}{2\eta_0}$.

$E_{t0} \cong 2E_{i0}\frac{\eta_2}{\eta_0}$, $\quad H_{t0} \cong \frac{2E_{i0}}{\eta_0}$ ⟶ $(\mathcal{P}_{av})_t = 2\frac{E_{i0}^2\alpha_2}{\eta_0^2\sigma_2}e^{-2\alpha_2 z}$.

$\therefore \frac{(\mathcal{P}_{av})_t}{(\mathcal{P}_{av})_i} = \frac{4\alpha_2}{\eta_0\sigma_2}e^{-2\alpha_2 z} = 1.054\times 10^{-3}e^{-0.795z}$

d) $20\log_{10} e^{-\alpha_2 z} = -30.$ ⟶ $z = \frac{1.5}{\alpha_2\log_{10}e} = 8.69 \ (m)$.

P.8-36



a) Snell's law:

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{1}{n},$$

$$\theta_t = \sin^{-1}\left(\frac{1}{n}\sin\theta_i\right).$$

b) $\cos\theta_t = \sqrt{1-\left(\frac{1}{n}\sin\theta_i\right)^2}$.

$$\ell_1 = \overline{BC} = \overline{AC}\tan\theta_t = d\frac{\sin\theta_t}{\cos\theta_t} = \frac{d\sin\theta_i}{\sqrt{n^2-\sin^2\theta_i}}.$$

c) $\ell_2 = \overline{BD} = \overline{AC}\sin(\theta_i-\theta_t) = \frac{d}{\cos\theta_t}(\sin\theta_i\cos\theta_t - \cos\theta_i\sin\theta_t)$

$= d\sin\theta_i\left[1 - \frac{\cos\theta_i}{\sqrt{n^2-\sin^2\theta_i}}\right]$.

P.8-37

a) $\sin\theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$ ⟶ $\sin\theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i > 1$ for $\theta_i > \theta_c$.

$\cos\theta_t = -j\sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right)\sin^2\theta_i - 1}$.

From Eqs. (8-200) and (8-201):

$$\bar{E}_t(x,z) = \bar{a}_y E_{t0} e^{-\alpha_2 z} e^{-j\beta_{2x} x},$$

$$\bar{H}_t(x,z) = \frac{E_{t0}}{\eta_2}\left(\bar{a}_x j\alpha_2 + \bar{a}_z\sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i\right) e^{-\alpha_2 z} e^{-j\beta_{2x} x},$$

where $\beta_{2x} = \beta_2 \sin\theta_t = \beta_2\sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i,$

$$\alpha_2 = \beta_2\sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right)\sin^2\theta_i - 1},$$

$$E_{t0} = \frac{2\eta_2\cos\theta_i E_{i0}}{\eta_2\cos\theta_i - j\eta_1\sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right)\sin^2\theta_i - 1}} \quad \text{from Eq. (8-207).}$$

b) $(\mathcal{P}_{av})_{2z} = \frac{1}{2}\mathcal{R}e\,(E_{ty} H_{tx}^*) = 0.$

P. 8-38 Given $\theta_i = \theta_c \longrightarrow \theta_t = \pi/2$, $\cos\theta_t = 0$.

a) From Eq. (8-207): $(E_{t0}/E_{i0})_\perp = 2.$

b) From Eq. (8-221): $(E_{t0}/E_{i0})_\parallel = 2\eta_2/\eta_1.$

c) $\bar{E}_i(x,z;t) = \bar{a}_y E_{i0}\cos\omega\left[t - \frac{n_1}{c}(x\sin\theta_i + z\cos\theta_i)\right],$

$$\bar{E}_t(x,z;t) = \bar{a}_y 2E_{i0}e^{-\alpha z}\cos\omega(t - \frac{n_2}{c}x\sin\theta_t)$$

$$= \bar{a}_y 2E_{i0}e^{-\alpha z}\cos\omega(t - \frac{n_1}{c}x\sin\theta_i),$$

where $\alpha = \frac{n_2\omega}{c}\sqrt{\left(\frac{n_1}{n_2}\sin\theta_i\right)^2 - 1} = 0$ when $\theta = \theta_c$.

P. 8-39. a) $\theta_c = \sin^{-1}\sqrt{\epsilon_{r2}/\epsilon_{r1}} = \sin^{-1}\sqrt{1/81} = 6.38°.$

b) $\theta_i = 20° > \theta_c.$ $\sin\theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i = 3.08,$ $\cos\theta_t = -j2.91.$

$$\Gamma_\perp = \frac{\sqrt{\epsilon_{r1}}\cos\theta_i - \cos\theta_t}{\sqrt{\epsilon_{r1}}\cos\theta_i + \cos\theta_t} = e^{j38°} = e^{j0.66}$$

c) $$\tau_\perp = \frac{2\sqrt{\epsilon_{r1}}\cos\theta_i}{\sqrt{\epsilon_{r1}}\cos\theta_i + \cos\theta_t} = 1.89\,e^{j19°} = 1.89\,e^{j0.33}$$

d) The transmitted wave in air varies as $e^{-\alpha_2 z}e^{-j\beta_{2x}x}$,

Where $\alpha_2 = \beta_2\sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right)\sin^2\theta_i - 1} = \frac{2\pi}{\lambda_0}(2.91).$

Attenuation in air for each wavelength

$= 20\log_{10} e^{-\alpha_2\lambda_0} = 159$ (dB).

P.8-40 When the incident light first strikes the hypotenuse surface, $\theta_i = \theta_t = 0$, $\tau_1 = \frac{2\eta_2}{\eta_2 + \eta_0}$.

$$\frac{(\mathcal{P}_{av})_{t1}}{(\mathcal{P}_{av})_i} = \frac{\eta_0}{\eta_2}\tau_1^2 = \frac{4\eta_0\eta_2}{(\eta_2+\eta_0)^2}.$$

Total reflections occur inside the prism at both slanting surfaces because

$$\theta_i = 45° > \theta_c = \sin^{-1}\left(\frac{1}{2}\right) = 30°.$$

On exit from the prism, $\tau_2 = \frac{2\eta_0}{\eta_2 + \eta_0}$.

$$\frac{(\mathcal{P}_{av})_o}{(\mathcal{P}_{av})_{t1}} = \frac{\eta_2}{\eta_0}\tau_2^2 = \frac{4\eta_0\eta_2}{(\eta_2+\eta_0)^2}.$$

$$\therefore \frac{(\mathcal{P}_{av})_o}{(\mathcal{P}_{av})_i} = \left[\frac{4\eta_0\eta_2}{(\eta_2+\eta_0)^2}\right]^2 = \left[\frac{4\sqrt{\epsilon_r}}{(1+\sqrt{\epsilon_r})^2}\right]^2 = 0.79.$$

P.8-41 a) $\eta_0 \sin\theta_a = n_1 \sin(90° - \theta_c) = n_1 \cos\theta_c$

$$= n_1\sqrt{1-\sin^2\theta_c} = n_1\sqrt{1-(n_2/n_1)^2} = \sqrt{n_1^2 - n_2^2}.$$

$$\sin\theta_a = \frac{1}{n_0}\sqrt{n_1^2-n_2^2} = \sqrt{n_1^2-n_2^2}. \qquad (n_0 = 1)$$

b) $N.A. = \sin\theta_a = \sqrt{2^2 - 1.74^2} = 0.9861$,

$$\theta_a = \sin^{-1}0.9861 = 80.4°.$$

P.8-42 Eq. (8-185): $\frac{\eta_2}{\eta_1} = \frac{\sin\theta_t}{\sin\theta_i}$.

a) Eq. (8-206): $$\Gamma_\perp = \frac{(\eta_2/\eta_1)\cos\theta_i - \cos\theta_t}{(\eta_2/\eta_1)\cos\theta_i + \cos\theta_t} = \frac{\sin\theta_t\cos\theta_i - \cos\theta_t\sin\theta_i}{\sin\theta_t\cos\theta_i + \cos\theta_t\sin\theta_i} = \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)}.$$

Eq. (8-207): $$\tau_\perp = \frac{2(\eta_2/\eta_1)\cos\theta_i}{(\eta_2/\eta_1)\cos\theta_i + \cos\theta_t} = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_t+\theta_i)}.$$

b) Eq. (8-221): $$\Gamma_\parallel = \frac{(\eta_2/\eta_1)\cos\theta_t - \cos\theta_i}{(\eta_2/\eta_1)\cos\theta_t + \cos\theta_i} = \frac{\sin\theta_t\cos\theta_t - \sin\theta_i\cos\theta_i}{\sin\theta_t\cos\theta_t + \sin\theta_i\cos\theta_i} = \frac{\sin 2\theta_t - \sin 2\theta_i}{\sin 2\theta_t + \sin 2\theta_i}.$$

Eq. (8-222): $$\tau_\parallel = \frac{2(\eta_2/\eta_1)\cos\theta_i}{(\eta_2/\eta_1)\cos\theta_t + \cos\theta_i} = \frac{4\sin\theta_t\cos\theta_i}{\sin 2\theta_t + \sin 2\theta_i}.$$

P.8-43 a) For perpendicular polarization and $\mu_1 \neq \mu_2$:

$$\sin\theta_{B\perp} = \frac{1}{\sqrt{1+\left(\frac{\mu_1}{\mu_2}\right)}}.$$

Under condition of no reflection:

$$\cos\theta_t = \sqrt{1-\frac{n_1^2}{n_2^2}\sin^2\theta_{B\perp}} = \frac{1}{\sqrt{1+\left(\frac{\mu_2}{\mu_1}\right)}}$$

$$= \sin\theta_{B\perp} \longrightarrow \theta_t + \theta_{B\perp} = \pi/2.$$

b) For parallel polarization and $\epsilon_1 \neq \epsilon_2$:

$$\sin\theta_{B\parallel} = \frac{1}{\sqrt{1+\left(\frac{\epsilon_1}{\epsilon_2}\right)}}.$$

$$\cos\theta_t = \sqrt{1-\frac{n_1^2}{n_2^2}\sin^2\theta_{B\parallel}} = \frac{1}{\sqrt{1+\left(\frac{\epsilon_2}{\epsilon_1}\right)}}$$

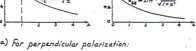
$$= \sin\theta_{B\parallel} \longrightarrow \theta_t + \theta_{B\parallel} = \pi/2.$$

P.8-44 a) $\sin\theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$; $\sin\theta_{B\parallel} = \frac{1}{\sqrt{1+\left(\frac{\epsilon_1}{\epsilon_2}\right)}}$

$$\longrightarrow \tan\theta_{B\parallel} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}.$$

$$\therefore \sin\theta_c = \tan\theta_{B\parallel}. \quad (\theta_c > \theta_{B\parallel})$$

b) Let $\epsilon_1/\epsilon_2 = x$.



P.8-45 a) For perpendicular polarization:

$$\Gamma_\perp = \frac{\sqrt{\epsilon_{r1}}\cos\theta_i - \sqrt{\epsilon_{r2}}\cos\theta_t}{\sqrt{\epsilon_{r1}}\cos\theta_i + \sqrt{\epsilon_{r2}}\cos\theta_t},$$

$$\sin\theta_t = \sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\sin\theta_i, \quad \cos\theta_t = \sqrt{1-\left(\frac{\epsilon_{r1}}{\epsilon_{r2}}\right)\sin^2\theta_i}.$$

$$\Gamma_\perp = \frac{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\cos\theta_i - \sqrt{1-\frac{\epsilon_{r1}}{\epsilon_{r2}}\sin^2\theta_i}}{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\cos\theta_i + \sqrt{1-\frac{\epsilon_{r1}}{\epsilon_{r2}}\sin^2\theta_i}}.$$

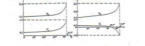
$$\tau_\perp = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = \frac{2\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\cos\theta_i}{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\cos\theta_i + \sqrt{1-\frac{\epsilon_{r1}}{\epsilon_{r2}}\sin^2\theta_i}}.$$

For parallel polarization:

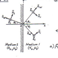
$$\Gamma_{\parallel} = \frac{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\sqrt{1-\left(\frac{\epsilon_{r1}}{\epsilon_{r2}}\right)\sin^2\theta_i} - \cos\theta_i}{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\sqrt{1-\left(\frac{\epsilon_{r1}}{\epsilon_{r2}}\right)\sin^2\theta_i} + \cos\theta_i},$$

$$\tau_{\parallel} = \frac{2\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\cos\theta_i}{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\sqrt{1-\left(\frac{\epsilon_{r1}}{\epsilon_{r2}}\right)\sin^2\theta_i} + \cos\theta_i}.$$

b) $\epsilon_{r1}/\epsilon_{r2} = 2.25$, $\sqrt{\epsilon_{r1}/\epsilon_{r2}} = 1.5 \longrightarrow \theta_c = \sin^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}} = 41.8°$.



P.8-46



Given: $\bar{E}_i(x,z) = \bar{a}_y E_{i0} e^{-jk_0(x\sin\theta_i - z\cos\theta_i)}$,

$\bar{a}_{ni} = \bar{a}_x \sin\theta_i - \bar{a}_z \cos\theta_i$.

$\bar{H}_i(x,z) = \frac{1}{\eta_0}\bar{a}_{ni}\times\bar{E}_i(x,z)$

$= \frac{1}{\eta_0}(\bar{a}_x\cos\theta_i + \bar{a}_z\sin\theta_i)e^{-jk_0(x\sin\theta_i - z\cos\theta_i)}$.

$\epsilon_c = \epsilon' - j\epsilon''$, $k_2 = \omega\sqrt{\mu_0\epsilon_c} = \omega\sqrt{\mu_0\epsilon_0\left(\frac{\epsilon'}{\epsilon_0} - j\frac{\epsilon''}{\epsilon_0}\right)}$

$= k_0\sqrt{\epsilon_r' - j\epsilon_r''}$.

a) From Eq. (8-207):

$$\tau_{\perp} = \frac{2(\eta_2/\eta_0)\cos\theta_i}{(\eta_2/\eta_0)\cos\theta_i + \cos\theta_t},$$

where $(\eta_2/\eta_0) = \sqrt{\epsilon_0/\epsilon_c} = 1/\sqrt{\epsilon_r' - j\epsilon_r''}$.

$\bar{E}_t(x,z) = \bar{a}_y \tau_{\perp} E_{i0} e^{-jk_2(x\sin\theta_t - z\cos\theta_t)}$,

$\bar{H}_t(x,z) = \frac{1}{\eta_2}\bar{a}_{nt}\times\bar{E}_t(x,z) = \frac{1}{\eta_2}(\bar{a}_x\cos\theta_t + \bar{a}_z\sin\theta_t)\tau_{\perp}\bar{E}_t(x,z)$.

b) From Eq. (8-185): $\sin\theta_t = \dfrac{\sin\theta_i}{\sqrt{\epsilon_r' - j\epsilon_r''}}$ (complex).

$$\cos\theta_t = \sqrt{1-\sin^2\theta_t} \quad \text{(complex)}.$$

The x- and z-components of $\bar{H}_t(x,z)$ in part a) have different amplitudes and are out of phase, indicating that it is elliptically polarized.

P.8-47

a) $$\Gamma_{\parallel}' = \left.\frac{(E_r)_{tan}}{(E_i)_{tan}}\right|_{z=0} = \frac{E_{r0}\cos\theta_i}{E_{i0}\cos\theta_i} = \frac{E_{r0}}{E_{i0}} = \Gamma_{\parallel} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i},$$

$$\tau_{\parallel}' = \left.\frac{(E_t)_{tan}}{(E_i)_{tan}}\right|_{z=0} = \frac{E_{t0}\cos\theta_t}{E_{i0}\cos\theta_i} = \tau_{\parallel}\left(\frac{\cos\theta_t}{\cos\theta_i}\right) = \frac{2\eta_1\cos\theta_t}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}.$$

b) From part a) we have

$$1 + \Gamma_{\parallel}' = \tau_{\parallel}'$$

This compares with

$$1 - \Gamma_{\parallel} = \tau_{\parallel}\left(\frac{\cos\theta_t}{\cos\theta_i}\right) \quad \text{in Eq. (8-223).}$$

Chapter 9

Theory and Applications of Transmission Lines

P.9-1

$$\bar{\nabla}\times\bar{H} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -j\beta \\ H_x & 0 & 0 \end{vmatrix} = \bar{a}_y j\omega\epsilon E_y \longrightarrow \frac{\partial H_x}{\partial y} = 0.$$

$$\bar{\nabla}\times\bar{E} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -j\beta \\ 0 & E_y & 0 \end{vmatrix} = -\bar{a}_x j\omega\mu H_x \longrightarrow \frac{\partial E_y}{\partial x} = 0.$$

P.9-2 a) $\bar{\nabla}\times(\bar{a}_x E_x + \bar{a}_y E_y) = -j\omega\mu(\bar{a}_x H_x + \bar{a}_y H_y)$.

$$\longrightarrow \begin{cases} \beta E_y = -\omega\mu H_x, & ① \\ \beta E_x = \omega\mu H_y. & ② \\ \frac{\partial E_y}{\partial x} = \frac{\partial E_x}{\partial y}. & ③ \end{cases}$$

$\nabla\times(a_x H_x + a_y H_y) = j\omega\epsilon(a_x E_x + a_y E_y)$.

$$\longrightarrow \begin{cases} \beta H_y = \omega\epsilon E_x, & ④ \\ \beta H_x = -\omega\epsilon E_y. & ⑤ \\ \frac{\partial H_y}{\partial x} = \frac{\partial H_x}{\partial y}. & ⑥ \end{cases}$$

From ② and ④: $\beta = \omega\sqrt{\mu\epsilon}$. ⑦

From ② or ④: $\frac{E_x}{H_y} = \sqrt{\frac{\mu}{\epsilon}} = \eta$. ⑧

From ① or ⑤: $\frac{E_y}{H_x} = -\sqrt{\frac{\mu}{\epsilon}} = -\eta$. ⑨

b) From ③: $\frac{\partial^2 E_y}{\partial y\,\partial x} = \frac{\partial^2 E_x}{\partial y^2}$. ⑩

From ②, ④, and ⑤: $\frac{\partial E_x}{\partial x} = -\frac{\partial E_y}{\partial y} \longrightarrow \frac{\partial^2 E_x}{\partial x^2} = -\frac{\partial^2 E_y}{\partial x\,\partial y}$. ⑪

Combining ⑩ and ⑪, we have $\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} = 0$.

Similarly, $\frac{\partial^2 H_x}{\partial x^2} + \frac{\partial^2 H_x}{\partial y^2} = 0$.

P.9-3 Eq. (9-20): $Z_0 = \frac{d}{w}\sqrt{\frac{\mu}{\epsilon}}$.

a) $Z_0 = \frac{d'}{w}\sqrt{\frac{\mu}{2\epsilon}} = \frac{d}{w}\sqrt{\frac{\mu}{\epsilon}} \longrightarrow d' = \sqrt{2}\,d$.

b) $Z_0 = \frac{d}{w'}\sqrt{\frac{\mu}{2\epsilon}} = \frac{d}{w}\sqrt{\frac{\mu}{\epsilon}} \longrightarrow w' = \frac{1}{\sqrt{2}}w$.

c) $Z_0 = \frac{2d}{w}\sqrt{\frac{\mu}{\epsilon}} = \frac{d}{w'}\sqrt{\frac{\mu}{\epsilon}} \longrightarrow w' = 2w.$

d) $u_p = \frac{1}{\sqrt{\mu\epsilon}} \longrightarrow$ $u_{pa} = u_p/\sqrt{2}$ for case a.
$u_{pb} = u_p/\sqrt{2}$ for case b.
$u_{pc} = u_p$ for case c.

P.9-4 Given: $\sigma_c = 1.6\times10^7$ (S/m), $w = 0.02$ (m), $d = 2.5\times10^{-3}$ (m).
Lossy dielectric slab: $\mu = \mu_0$, $\epsilon_r = 3$, $\sigma = 10^{-3}$ (S/m).
$f = 5\times10^8$ (Hz).

a) $R = \frac{2}{w}\sqrt{\frac{\pi f\mu_c}{\sigma_c}} = 1.11$ (Ω/m)

$L = \mu\frac{d}{w} = 0.157$ (μH/m)

$G = \sigma\frac{w}{d} = 0.008$ (S/m)

$C = \epsilon\frac{w}{d} = 0.212$ (nF/m).

b) $\frac{|E_z|}{|E_y|} = \sqrt{\frac{\omega\epsilon}{\sigma_c}} = 4.167\times10^{-5}.$

c) $\omega L = 493.5 \gg R$, $\omega C = 0.667 \gg G$

$\gamma = j\omega\sqrt{LC}\left[1 + \frac{1}{2j}\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)\right] = 0.129 + j18.14$ (m^{-1})

$Z_0 = \sqrt{\frac{L}{C}}\left[1 + \frac{1}{2j}\left(\frac{R}{\omega L} - \frac{G}{\omega C}\right)\right] = 27.21 + j0.13$ (Ω).

P.9-5 Substituting Eqs. (9-38a) and (9-38b)

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

into Eq. (9-35a), we have

$$[\gamma V_0^+ - (R+j\omega L)I_0^+]e^{-\gamma z} - [\gamma V_0^- + (R+j\omega L)I_0^-]e^{\gamma z} = 0.$$

which requires

$$\gamma V_0^+ - (R+j\omega L)I_0^+ = 0$$

$$\gamma V_0^- + (R+j\omega L)I_0^- = 0.$$

$$\therefore \quad \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \frac{R+j\omega L}{\gamma}.$$

P.9-6 $\epsilon = \epsilon' - j\epsilon'' = \epsilon'\left(1 - j\frac{\epsilon''}{\epsilon'}\right) = \epsilon'\left(1 - j\frac{\sigma}{\omega\epsilon'}\right).$

From Eq. (9-70), $\gamma = \alpha + j\beta = j\omega\sqrt{\mu\epsilon'}\left(1 - j\frac{\epsilon''}{\epsilon'}\right)^{1/2}.$

Squaring both sides, we obtain two equations from the real and imaginary parts:

$$\alpha^2-\beta^2=-\omega^2\mu\epsilon'$$
$$2\alpha\beta=\omega^2\mu\epsilon'\left(\frac{\epsilon''}{\epsilon'}\right),$$

from which Eqs. (9-214) and (9-215) follow.

P.9-7 $\gamma=j\omega\sqrt{LC}\left(1-j\frac{R}{\omega L}\right)^{1/2}\left(1-j\frac{G}{\omega C}\right)^{1/2}$

$$=j\omega\sqrt{LC}\left[1-j\frac{R}{2\omega L}+\frac{1}{8}\left(\frac{R}{\omega L}\right)^2+j\frac{R^3}{16\omega^3L^3}\right]$$
$$\times\left[1-j\frac{G}{2\omega C}+\frac{1}{8}\left(\frac{G}{\omega C}\right)^2+j\frac{G^3}{16\omega^3C^3}\right]=\alpha+j\beta.$$

$\longrightarrow$ $\alpha=\frac{\sqrt{LC}}{2}\left(\frac{R}{L}+\frac{G}{C}\right)\left[1-\frac{1}{8\omega^2}\left(\frac{R}{L}-\frac{G}{C}\right)^2\right].$

$\beta=\omega\sqrt{LC}\left[1+\frac{1}{8\omega^2}\left(\frac{R}{L}-\frac{G}{C}\right)^2\right].$

$Z_0=\sqrt{\frac{L}{C}}\left(1-j\frac{R}{\omega L}\right)^{1/2}\left(1-j\frac{G}{\omega C}\right)^{-1/2}$

$$=\sqrt{\frac{L}{C}}\left[1-j\frac{R}{2\omega L}+\frac{1}{8}\left(\frac{R}{\omega L}\right)^2\right]\left[1-j\frac{G}{2\omega C}+\frac{3}{8}\left(\frac{G}{\omega C}\right)^2\right]=R_0+jX_0.$$

$\longrightarrow$ $R_0=\sqrt{\frac{L}{C}}\left[1+\frac{1}{8\omega^2}\left(\frac{R}{L}-\frac{G}{C}\right)\left(\frac{R}{L}+\frac{3G}{C}\right)\right].$

$X_0=-\frac{1}{2\omega}\sqrt{\frac{L}{C}}\left(\frac{R}{L}-\frac{G}{C}\right).$

$u_p=\frac{\omega}{\beta}\cong\frac{1}{\sqrt{LC}}\left[1-\frac{1}{8\omega^2}\left(\frac{R}{L}-\frac{G}{C}\right)^2\right].$

P.9-8 $\gamma=\sqrt{(R+j\omega L)(G+j\omega C)}=\sqrt{RG}\left(1+\frac{j\omega L}{R}\right)^{1/2}\left(1+\frac{j\omega C}{G}\right)^{1/2}=\alpha+j\beta.$

$\longrightarrow$ $\alpha\cong\sqrt{RG}\left[1+\frac{1}{8}\left(\frac{\omega L}{R}-\frac{\omega C}{G}\right)^2\right],\quad\beta\cong\frac{\omega}{2}\sqrt{RG}\left(\frac{L}{R}+\frac{C}{G}\right).$

$Z_0=\sqrt{\frac{R+j\omega L}{G+j\omega C}}=\sqrt{\frac{R}{G}}\left(1+j\frac{\omega L}{R}\right)^{1/2}\left(1+j\frac{\omega C}{G}\right)^{-1/2}=R_0+jX_0.$

$\longrightarrow$ $R_0\cong\sqrt{\frac{R}{G}}\left\{1+\frac{\omega^2}{8}\left[\left(\frac{L}{R}\right)^2+\frac{2LC}{RG}-3\left(\frac{C}{G}\right)^2\right]\right\}.$

$X_0=\frac{\omega}{2}\sqrt{\frac{R}{G}}\left(\frac{L}{R}-\frac{C}{G}\right).$

P.9-9 $Z_0=\sqrt{\frac{R+j\omega L}{G+j\omega C}}.\quad \mathcal{I}m(Z_0)=0.\longrightarrow\frac{R}{L}=\frac{G}{C}=k.$

From Eqs. (9-65), (9-64) and (9-66): $\alpha=R\sqrt{\frac{C}{L}},\quad\beta=\omega\sqrt{LC},\quad Z_0=\sqrt{\frac{L}{C}}.$

Given: $Z_0 = 50 + j0$ (Ω),

$\alpha = 0.01$ (dB/m) $= 0.00115$ (Np/m),

$\beta = 0.8\pi$ (rad/m),

$f = 10^8$ (Hz).

$R = \alpha Z_0 = 0.0576$ (Ω/m), $\quad L = \frac{\beta Z_0}{2\pi f} = 0.20$ $(\mu H/m)$,

$G = \frac{\alpha^2}{R} = 23.$ $(\mu S/m)$, $\quad C = \frac{L}{Z_0^2} = 80$ (pF/m).

P.9-10 a) For two-wire transmission line:

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{1}{\pi}\sqrt{\frac{\mu}{\epsilon}}\cosh^{-1}\left(\frac{D}{2a}\right) = \frac{120}{\sqrt{\epsilon_r}}\ln\left[\frac{D}{2a} + \sqrt{\left(\frac{D}{2a}\right)^2 - 1}\right] = 300\ (\Omega).$$

$$\frac{D}{2a} = 21.27 \longrightarrow D = 25.5 \times 10^{-3}\ (m).$$

b) For coaxial transmission line:

$$Z_0 = \frac{1}{2\pi}\sqrt{\frac{\mu}{\epsilon}}\ln\left(\frac{b}{a}\right) = \frac{60}{\sqrt{\epsilon_r}}\ln\left(\frac{b}{a}\right) = 75.$$

$$\frac{b}{a} = 6.52 \longrightarrow b = 3.91 \times 10^{-3}\ (m).$$

P.9-11 $(P_{av})_L = (P_{av})_i = \frac{1}{2}\mathcal{R}e\left[V_i I_i^*\right]$ $\qquad V_i = \frac{Z_i}{Z_g + Z_i}V_g$,

$$= \frac{|V_g|^2 R_i}{(R_g + R_i)^2 + (X_g + X_i)^2}. \qquad I_i = \frac{V_g}{Z_g + Z_i}.$$

To maximize $(P_{av})_L$, set $\frac{\partial (P_{av})_L}{\partial R_i} = 0$, and $\frac{\partial (P_{av})_L}{\partial X_i} = 0$. $\Big\}$ $R_i = R_g$, $X_i = -X_g$, or $Z_i = Z_g^*$.

$$\text{Max. } (P_{av})_L = \frac{|V_g|^2}{4R_g} = (P_{av})_{Z_g}.$$

$\longrightarrow$ Max. power-transfer efficiency $= 50\%$.

P.9-12 $V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$.

$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$.

At $z = 0$: $V(0) = V_i = V_0^+ + V_0^-$; $\quad I(0) = I_i = I_0^+ + I_0^- = \frac{1}{Z_0}(V_0^+ - V_0^-)$.

$$\longrightarrow V_0^+ = \frac{1}{2}(V_i + I_i Z_0), \quad V_0^- = \frac{1}{2}(V_i - I_i Z_0).$$

a) $V(z) = \frac{1}{2}(V_i + I_i Z_0)e^{-\gamma z} + \frac{1}{2}(V_i - I_i Z_0)e^{\gamma z}$.

$I(z) = \frac{1}{2Z_0}(V_i + I_i Z_0)e^{-\gamma z} - \frac{1}{2Z_0}(V_i - I_i Z_0)e^{\gamma z}$.

b) $V(z) = V_i\cosh\gamma z - I_i Z_0\sinh\gamma z$,

$I(z) = I_i\cosh\gamma z - \frac{V_i}{Z_0}\sinh\gamma z$.

P.9-13 From Fig. 9-44(b):
$$V_1 = \frac{Z_1}{2}I_1 + \frac{1}{Y_2}(I_1 - I_2) = \left(\frac{Z_1}{2} + \frac{1}{Y_2}\right)I_1 - \frac{1}{Y_2}I_2. \quad ①$$

Also:
$$I_1 = I_2 + \left(V_2 + \frac{Z_1}{2}I_2\right)Y_2 = Y_2 V_2 + \left(1 + \frac{Z_1 Y_2}{2}\right)I_2. \quad ②$$

Substituting ② in ①:
$$V_1 = \left(1 + \frac{Z_1 Y_2}{2}\right)V_2 + Z_1\left(1 + \frac{Z_1 Y_2}{4}\right)I_2. \quad ③$$

a) Setting $z' = \ell$, $I_L = I_2$, and $I_L Z_L = V_2$ in Eqs.(9-100a) and (9-100b) on p.451:
$$V_1 = V(\ell) = (\cosh\gamma\ell)V_2 + (Z_0 \sinh\gamma\ell)I_2, \quad ④$$
$$I_1 = I(\ell) = \left(\frac{1}{Z_0}\sinh\gamma\ell\right)V_2 + (\cosh\gamma\ell)I_2. \quad ⑤$$

Both Eqs. ③ & ② and Eqs. ④ & ⑤ are of the following forms:
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} V_2 \\ I_2 \end{bmatrix}, \quad ⑥$$

where
$$A = D = 1 + \frac{Z_1 Y_2}{2} = \cosh\gamma\ell, \quad ⑦$$
$$B = Z_1\left(1 + \frac{Z_1 Y_2}{4}\right) = Z_0 \sinh\gamma\ell, \quad ⑧$$
and
$$C = Y_2 = \frac{1}{Z_0}\sinh\gamma\ell. \quad ⑨$$
$$\longrightarrow AD - BC = \cosh^2\gamma\ell - \sinh^2\gamma\ell = 1.$$

b) ⑨ is the required result in Eq.(9-220).
Eq.(9-219) can be obtained by using ⑨ in ⑦:
$$Z_1 = \frac{1}{Y_2}(2A - 1) = 2Z_0 \tanh\frac{\gamma\ell}{2}.$$

P.9-14 a) $-\frac{dV}{dz} = RI, \quad -\frac{dI}{dz} = GV.$
$$\begin{cases} \frac{d^2V}{dz^2} = RGV, \\ \frac{d^2I}{dz^2} = RGI. \end{cases}$$

b) $V(z) = V_0^+ e^{-\alpha z} + V_0^- e^{\alpha z}$

$I(z) = I_0^+ e^{-\alpha z} + I_0^- e^{\alpha z}, \quad \alpha = \sqrt{RG}$

$$\frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = R_0 = \sqrt{\frac{R}{G}}.$$

We have $V(z) = \frac{1}{2}(V_i + I_i R_0)e^{-\alpha z} + \frac{1}{2}(V_i - I_i R_0)e^{\alpha z}$

$$I(z) = \frac{1}{2}\left(\frac{V_i}{R_0} + I_i\right)e^{-\alpha z} - \frac{1}{2}\left(\frac{V_i}{R_0} - I_i\right)e^{\alpha z},$$

where $V_i = \frac{R_i}{R_g + R_i} V_g$ and $I_i = \frac{V_g}{R_g + R_i}$.

c) For an infinite line, $R_i = R_0$:

$$V(z) = \frac{R_0}{R_g + R_0} V_g e^{-\alpha z}, \qquad I(z) = \frac{V_g}{R_g + R_0} e^{-\alpha z}.$$

d) For a finite line of length ℓ terminated in R_L:

$$R_i = R_0 \frac{R_L + R_0 \tanh \alpha\ell}{R_0 + R_L \tanh \alpha\ell}.$$

P.9-15 Distortionless line: $R_0 = \sqrt{\frac{L}{C}} = 50\ (\Omega)$, $R = 0.5\ (\Omega/m)$

$$\tan\left(\frac{R}{\omega L}\right) = \tan\left(\frac{G}{\omega C}\right) = 0.0018.$$

$$\longrightarrow \frac{G}{\omega C} = 0.0018;\quad \frac{G}{C} = 8000\pi \times 0.0018 = 45.2 = \frac{R}{L}.$$

$$L = \frac{R}{G/C} = 0.011\ (H/m), \qquad C = \frac{L}{R_0^2} = 4.42\ (\mu F/m).$$

$$\alpha = \frac{R}{R_0} = 0.010\ (Np/m). \qquad \beta = \omega\sqrt{LC} = 5.55\ (rad/m).$$

a) $V(z) = \frac{V_g R_0}{R_0 + R_g} e^{-\alpha z} e^{-j\beta z} = \frac{50}{9 + j3} e^{-0.01z} e^{-j5.55z}$; $I(z) = \frac{V(z)}{50}$.

$$\therefore V(z,t) = 5.27 e^{-0.01z} \sin(8000\pi t - 5.55z - 0.322) \quad (V),$$

$$I(z,t) = 0.105 e^{-0.01z} \sin(8000\pi t - 5.55z - 0.322) \quad (A).$$

b) At $z = 50\ (m)$: $V(50,t) = 3.20 \sin(8000\pi t - 0.432\pi)$ (V).

$$I(50,t) = 0.064 \sin(8000\pi t - 0.432\pi) \quad (A).$$

c) $(P_{av})_L = \frac{1}{2}\mathcal{Re}|V_L I_L^*| = \frac{1}{2}(3.20 \times 0.064) = 0.102\ (W)$.

P.9-16 a) From Eq.(9-117): $Z_{is} = Z_0 \tanh \gamma\ell \cong Z_0(\gamma\ell)$.

From Eqs.(9-37) and (9-41): $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$, $Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$.

$$\therefore Z_{is} = (R + j\omega L)\ell.$$

b) From Eq.(9-116): $Z_{io} = Z_0 \coth \gamma\ell \cong \frac{Z_0}{\gamma\ell} = \frac{G - j\omega C}{[G^2 + \omega^2 C^2]\ell}$.

P.9-17 a) From Eq. (9-117): $Z_{is} = Z_0 \tanh \gamma \ell = Z_0 \dfrac{1 - e^{-2\gamma \ell}}{1 + e^{-2\gamma \ell}}$.

For $\ell = \lambda/4$, $\beta \ell = \pi/2$, $\alpha\lambda/2 << 1$.

$$Z_{is} = Z_0 \frac{1 - e^{-2\alpha(\lambda/4)} e^{-j\pi}}{1 + e^{-2\alpha(\lambda/4)} e^{-j\pi}} \cong Z_0 \frac{1 + (1 - \alpha\lambda/2)}{1 - (1 - \alpha\lambda/2)}$$

$$\cong 4Z_0/\alpha\lambda.$$

b) From Eq. (9-116): $Z_{io} = Z_0 \coth \gamma \ell = Z_0 \dfrac{1 + e^{-2\gamma \ell}}{1 - e^{-2\gamma \ell}}$.

For $\ell = \lambda/4$, $Z_{io} = Z_0 \dfrac{1 + e^{-(\alpha\lambda/2)} e^{-j\pi}}{1 - e^{-(\alpha\lambda/2)} e^{-j\pi}} \cong Z_0 \dfrac{1 - (1 - \alpha\lambda/2)}{1 + (1 - \alpha\lambda/2)}$

$$\cong Z_0 \alpha\lambda/4.$$

P.9-18 $\beta \ell = \frac{2\pi f}{c} \ell = \frac{8\pi}{3} = 480°$.

$\tan \beta \ell = \tan 480° = -1.732$.

$$Z_i = Z_0 \frac{Z_L + jZ_0 \tan \beta \ell}{Z_0 + jZ_L \tan \beta \ell} = 50 \frac{(40 + j30) + j50(-1.732)}{50 + j(40 + j30)(-1.732)}$$

$$= 26.3 - j9.87 \quad (\Omega).$$

P.9-19 Given: $Z_{io} = Z_0 \coth \gamma \ell = 250 \angle -50° \quad (\Omega)$.

$Z_{is} = Z_0 \tan \gamma \ell = 360 \angle 20° \quad (\Omega)$.

a) $Z_0 = \sqrt{Z_{io} Z_{is}} = 300 \angle -15° = 289.8 - j77.6 \quad (\Omega)$.

$\tanh \gamma \ell = \sqrt{\dfrac{Z_{is}}{Z_{io}}} = 1.2 \angle 35° = 0.983 + j688 = \tanh(\alpha \ell + j\beta \ell)$.

$\ell = 4 \text{ (m)} \longrightarrow \alpha = 0.139 \text{ (Np/m)}$,

$\beta = 0.235 \text{ (rad/m)}$.

b) $Z_0 = \sqrt{\dfrac{R + j\omega L}{G + j\omega C}}$, $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$.

$\longrightarrow R + j\omega L = \gamma Z_0$; $G + j\omega C = \dfrac{\gamma}{Z_0}$.

$\omega = \beta c = 0.235 \times 3 \times 10^8 = 0.705 \times 10^8 \text{ (rad/m)}$.

We obtain: $R = 58.6 \, (\Omega)$, $L = 0.812 \, (\mu H/m)$,

$G = 0.246 \text{ (mS/m)}$, $C = 12.4 \text{ (pF/m)}$.

P.9-20 a) Since the line is very short compared to a wavelength, we may use Eqs. (9-111) and (9-112).

$$\therefore\ C = \frac{54\times10^{-12}}{0.6} = 9\times10^{-11}\ (F/m), \quad L = \frac{0.3\times10^{-6}}{0.6} = 5\times10^{-7}\ (H/m). \quad \Big\}\ R_0 = \sqrt{\frac{L}{C}} = 74.5\ (\Omega).$$

$$\mu\epsilon = LC \longrightarrow \epsilon_r = \frac{LC}{\mu_0\epsilon_0} = 4.05.$$

b) $\beta = \frac{\omega}{u_p} = 2\pi\times10^{8}\sqrt{LC} = 0.42\ (rad/m);\quad \beta\ell = 0.42\times0.6 = 0.25\ (rad) = 14.4^\circ$

$$\therefore\ X_{i0} = -R_0\cot\beta\ell = -\frac{1}{\omega C\ell} = -290\ (\Omega),$$

$$X_{is} = R_0\tan\beta\ell = \omega L\ell = 19.2\ (\Omega).$$

P.9-21 From Eq. (9-116): $Z_{io} = Z_0\coth\gamma\ell = Z_0\frac{\cosh(\alpha+j\beta)\ell}{\sinh(\alpha+j\beta)\ell}$

$$= Z_0\frac{\cosh\alpha\ell\cos\beta\ell + j\sinh\alpha\ell\sin\beta\ell}{\sinh\alpha\ell\cos\beta\ell + j\cosh\alpha\ell\sin\beta\ell}. \quad ①$$

For a low-loss line, $\alpha\ell \ll 1$, $\cosh\alpha\ell \cong 1$, $\sinh\alpha\ell \cong \alpha\ell$. $\quad Z_{io} \cong Z_0\frac{\cos\beta\ell + j\alpha\ell\sin\beta\ell}{\alpha\ell\cos\beta\ell + j\sin\beta\ell}. \quad ②$

At $\ell = n\lambda/2$, $\beta_0\ell = n\pi$. $\quad (Z_{io})_{max} \cong \frac{Z_0}{\alpha\ell}$ (antiresonance). $\quad ③$

When the frequency is slightly off antiresonance:

$f = f_0 + \Delta f\ (\Delta f \ll f_0)$, $\beta\ell = n\pi(1+\frac{\Delta f}{f_0})$, $\cos\beta\ell \cong \cos n\pi = (-1)^n$

and $\sin\beta\ell = \cos n\pi\sin(n\pi\Delta f/f_0) \cong (-1)^n n\pi(\Delta f/f_0)$.

② becomes, after leaving out second-order small terms:

$$Z_{io} \cong \frac{Z_0}{\alpha\ell + jn\pi(\Delta f/f_0)}. \quad ④$$

Combining ③ & ④: $\frac{|Z_{io}|^2}{(Z_{io})^2_{max}} = \frac{1}{1+[n\pi(\Delta f/f_0\alpha\ell)]^2}. \quad ⑤$

Half-power points at: $n\pi\left(\frac{\Delta f}{f_0\alpha\ell}\right) = \pm1$; or $\Delta f = \pm\frac{\alpha f_0\ell}{n\pi}. \quad ⑥$

For $\ell = n\lambda/2$, $f_0\ell = \frac{n}{2}f_0\lambda = \frac{n}{2\sqrt{LC}}$, and $\Delta f = \pm\frac{\alpha}{2\pi\sqrt{LC}}$,

which, for a low-loss transmission line, becomes (see Eq. 9-55): $\Delta f = \pm\frac{1}{4\pi}\left(\frac{R}{L}+\frac{G}{C}\right) \longrightarrow$ Half-power bandwidth $2\Delta f = \frac{1}{2\pi}\left(\frac{R}{L}+\frac{G}{C}\right)$

$$Q = \frac{f_0}{2\Delta f} = \frac{1}{[(R/\omega_0 L)+(G/\omega_0 C)]}.$$

P.9-22 a) For a lossless quarter-wave line section:

$$Z_i = \frac{R_0^2}{Z_L} = \frac{R_0^2}{R_L + jX_L} = \frac{R_0^2 R_L}{R_L^2 + X_L^2} - j\frac{R_0^2 X_L}{R_L^2 + X_L^2} = R_i' + jX_i'.$$

$$\longrightarrow R_i' = \frac{R_0^2 R_L}{R_L^2 + X_L^2} \quad ① ; \quad X_i' = -\frac{R_0^2 X_L}{R_L^2 + X_L^2}. \quad ②$$

(Resistance R_i' and capacitive reactance X_i' in series.)

Input impedance Z_i can also be expressed in terms of a resistance R_i and a capacitive reactance X_i in parallel:

$$Z_i = \frac{jX_i R_i}{R_i + jX_i} = \frac{R_i X_i^2}{R_i^2 + X_i^2} + j\frac{R_i^2 X_i}{R_i^2 + X_i^2} = R_i' + jX_i'. \quad ③$$

Combining Eqs. ①, ②, and ③, we find

$$R_i = \frac{R_0^2}{R_L} \quad \text{and} \quad X_i = -\frac{R_0^2}{X_L},$$

both of which are reminiscent of Eq. (9-114).

b) From Eq. (9-100a): $V(z') = I_L(Z_L \cos\beta z' + R_0 \sin\beta z')$.

At the input, $z' = \lambda/4$, $\beta z' = \pi/2$, we have

$$V_i = V(\lambda/4) = I_L R_0.$$

At the load, $z' = 0$, $\beta z' = 0$, and $V_L = V(0) = I_L Z_L$.

$$\therefore \quad \frac{|V_i|}{|V_L|} = \frac{R_0}{|Z_L|} = \frac{R_0}{\sqrt{R_L^2 + X_L^2}}.$$

P.9-23 a) $$|\Gamma| = \frac{S-1}{S+1} = \frac{\left|\frac{Z_L}{Z_0} - 1\right|}{\left|\frac{Z_L}{Z_0} + 1\right|} = \frac{\sqrt{(r_L - 1)^2 + x_L^2}}{\sqrt{(r_L + 1)^2 + x_L^2}},$$

where $r_L = R_L/Z_0$ and $x_L = X_L/Z_0$.

$$\longrightarrow x_L = \pm\left[\frac{\left(\frac{S-1}{S+1}\right)^2 (r_L + 1)^2 - (r_L - 1)^2}{1 - \left(\frac{S-1}{S+1}\right)^2}\right]^{1/2}.$$

When $S = 3$, $x_L = \pm\sqrt{(10 r_L - 3 r_L^2 - 3)/3}$.

b) $S = 3$ and $r_L = 150/75 = 2 \longrightarrow x_L = \pm\sqrt{5/3}$.

$$X_L = x_L Z_0 = \pm 96.8 \ (\Omega).$$

c) From Eq. (9-147): $r_L + jx_L = \dfrac{r_m + jt}{1 + jr_m t}$,

where $r_m = R_m/Z_0$, and $t = \tan \beta \ell_m$.

$\longrightarrow r_m = \dfrac{(1 + r_L^2 + x_L^2) \pm \sqrt{(1 + r_L^2 + x_L^2)^2 - 4r_L^2}}{2r_L}$

$= 3$ or $\frac{1}{3}$, for $r_L = 2$ and $x_L^2 = 5/3$.

Also, $x_L = \dfrac{(1 - r_m^2)t}{1 + r_m^2 t^2} \longrightarrow t = \dfrac{1}{2x_L r_m^2}\left[(1 - r_m^2) \pm \sqrt{(1 - r_m^2)^2 - 4x_L^2 r_m^2}\right]$

$r_m = 3$ yields negative t (discard).

For $r_m = \frac{1}{3}$, $t = \begin{cases} 3\sqrt{3/5} \longrightarrow \ell_m = 0.1865\lambda \\ \text{or } \sqrt{15} \longrightarrow \ell_m = 0.2098\lambda \end{cases}$

Use $\ell_m = 0.2095\lambda$ to obtain V_{min} nearest to the load at $(0.5 - 0.2098)\lambda = 0.2902\lambda$.

P.9-24 a) $|\Gamma|^2 = \left|\dfrac{(R_L - Z_0) + jX_L}{(R_L + Z_0) + jX_L}\right| = \dfrac{(R_L - Z_0)^2 + X_L^2}{(R_L + Z_0)^2 + X_L^2}$

$\dfrac{\partial |\Gamma|^2}{\partial Z_0} = 0 \longrightarrow Z_0 = \sqrt{R_L^2 + X_L^2}$.

If $Z_L = 40 + j30$ (Ω), $Z_0 = 50$ (Ω).

b) Min. $|\Gamma| = \sqrt{\dfrac{Z_0 - R_L}{Z_0 + R_L}} = \sqrt{\dfrac{50 - 40}{50 + 40}} = \dfrac{1}{3}$.

Min. $S = \dfrac{1 + \frac{1}{3}}{1 - \frac{1}{3}} = 2$.

c) From Eq. (9-147): $r_L + jx_L = \dfrac{r_m + jt}{1 + jr_m t} = 0.8 + j0.6$.

$\longrightarrow t = \dfrac{1}{2x_L r_m^2}\left[(1 - r_m^2) \pm \sqrt{(1 - r_m^2)^2 - 4x_L^2 r_m^2}\right]$ (See problem P.9-23.)

At voltage minimum, $r_m = \frac{1}{S} = \frac{1}{2}$.

$t = 1$ (Use negative sign.)

$\tan \beta \ell_m = \tan(2\pi \ell_m/\lambda) = 1 \longrightarrow \ell_m = \frac{\lambda}{8}$.

$\therefore$ Voltage minimum nearest to the load is $(\frac{\lambda}{2} - \frac{\lambda}{8})$ or $3\lambda/8$ from the load.

P.9-25 a) From Eqs. (9-133a) and (9-134):

$$V(z') = \frac{I_L}{2}(Z_L + Z_0)e^{\gamma z'}\left[1 + |\Gamma| e^{-2\alpha z'} e^{j\phi}\right],$$

where $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| e^{j\theta_\Gamma}$, $\quad \phi = \theta_\Gamma - 2\beta z'$.

$$\text{Max}|V(z')| = \left|\frac{I_L}{2}(Z_L + Z_0)e^{\alpha z'}\right|\left[1 + |\Gamma| e^{-2\alpha z'}\right] \text{ for } \phi = 0.$$

$$\text{min.}|V(z')| = \left|\frac{I_L}{2}(Z_L + Z_0)e^{\alpha z'}\right|\left[1 - |\Gamma| e^{-2\alpha z'}\right] \text{ for } \phi = \pi.$$

$$S(z') = \frac{\text{Max.}|V(z')|}{\text{min.}|V(z')|} = \frac{1 + |\Gamma| e^{-2\alpha z'}}{1 - |\Gamma| e^{-2\alpha z'}}.$$

(Slight ambiguity in z' — insignificant for small α.)

b) From Eq. (9-193): $Z_i(z') = \frac{1 + |\Gamma| e^{-2\alpha z'} e^{j\phi}}{1 - |\Gamma| e^{-2\alpha z'} e^{j\phi}} Z_0$.

At a voltage max., $\phi = 0$, $\quad Z_i(z') = S(z') Z_0$.

c) At a voltage min., $\phi = \pi$, $\quad Z_i(z') = \frac{Z_0}{S(z')}$.

P.9-26 From Eq. (9-147): $Z_i = R_0' \frac{Z_L + jR_0' t}{R_0' + jZ_L t} \longrightarrow Z_L = R_0' \frac{Z_i - jR_0' t}{R_0' - jZ_i t}$, $\; t = \tan\beta\ell'$

$$Z_i = R_0' \frac{Z_L + jR_0' t}{R_0' + jZ_L t} \longrightarrow Z_L = R_0' \frac{Z_i - jR_0' t}{R_0' - jZ_i t}.$$

With $Z_i = 50\ (\Omega)$ and $Z_L = 40 + j10\ (\Omega)$, we have

$$40 + j10 = R_0' \frac{50 - jR_0' t}{R_0' - j50t} \longrightarrow \begin{cases} 40R_0' + 500t = 50R_0', \\ 10R_0' - 2000t = -R_0'^2 t. \end{cases}$$

$\therefore R_0' = 38.7\ (\Omega)$; $\quad t = \tan\beta\ell' = 0.775 \longrightarrow \ell' = 0.105\lambda$.

P.9-27 a) $|\Gamma| = \frac{S-1}{S+1} = \frac{2-1}{2+1} = \frac{1}{3}$.

Eq. (9-133a): $V(z') = \frac{I_L}{2}(Z_L + Z_0)e^{j\beta z'}\left[1 + |\Gamma| e^{j\phi}\right]$.

Eq. (9-134): $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| e^{j\theta_\Gamma}$, $\quad \phi = \theta_\Gamma - 2\beta z'$.

Voltage is a minimum when $\phi = \pm\pi \longrightarrow \theta_\Gamma = 2\left(\frac{2\pi}{\lambda}\right) \times 0.3\lambda - \pi = 0.2\pi$.

$\therefore \Gamma = \frac{1}{3} e^{j0.2\pi}$.

b) $Z_L = Z_0\left(\frac{1+\Gamma}{1-\Gamma}\right) = 466 + j206\ (\Omega)$.

c) Terminating resistance $R_m = \frac{R_0}{S} = \frac{300}{2} = 150\ (\Omega)$.

$\ell_m = \frac{\lambda}{2} - z_m' = (0.5 - 0.3)\lambda = 0.2\lambda$.

Another set of solutions is: $R_m' = SR_0 = 600\ (\Omega)$ & $\ell_m' = 0.45\lambda$.

P.9-28 Eq. (9-147): $R_i + jX_i = R_0 \dfrac{R_m + jR_0 \tan\beta l_m}{R_0 + jR_m \tan\beta l_m}$.

Let $r_i = \dfrac{R_i}{R_0}$, $x_i = \dfrac{X_i}{R_0}$, $r_m = \dfrac{R_m}{R_0}$, and $t = \tan\beta l_m$.

$$r_i + jx_i = \frac{r_m + jt}{1 + jr_m t} \longrightarrow \begin{cases} r_m(1 + x_i t) = r_i, \\ t(1 - r_m r_i) = x_i. \end{cases}$$

We have

$$r_m = \frac{1}{2r_i}\left[(1 + r_i^2 + x_i^2) \pm \sqrt{(1 + r_i^2 + x_i^2)^2 - 4r_i^2}\right]$$

$$t = \frac{1}{2x_i}\left\{-\left[1 - (r_i^2 + x_i^2)\right] \pm \sqrt{\left[1 - (r_i^2 + x_i^2)\right]^2 + 4x_i^2}\right\},$$

$$l_m = \frac{\lambda}{2\pi}\tan^{-1} t.$$

P.9-29 $Z_L = Z_0 \dfrac{1 + \Gamma}{1 - \Gamma}$

$\Gamma = |\Gamma| e^{j\theta_\Gamma}$, $|\Gamma| = \dfrac{S - 1}{S + 1}$, $\theta_\Gamma = \dfrac{4\pi}{\lambda} z_m' \pm \pi$.

$$\therefore Z_L = Z_0 \frac{(S+1) - (S-1)\, e^{j(4\pi z_m'/\lambda)}}{(S+1) + (S-1)\, e^{j(4\pi z_m'/\lambda)}}$$

$$= Z_0 \frac{(S+1)\, e^{-j(2\pi z_m'/\lambda)} - (S-1)\, e^{j(2\pi z_m'/\lambda)}}{(S+1)\, e^{-j(2\pi z_m'/\lambda)} + (S-1)\, e^{j(2\pi z_m'/\lambda)}}$$

$$= Z_0 \frac{1 - jS\tan(2\pi z_m'/\lambda)}{S - j\tan(2\pi z_m'/\lambda)}.$$

P.9-30 a) Given: $V_g = 0.1\angle 0^\circ$ (V), $Z_g = Z_0 = 50$ (Ω), $R_L = 25$ (Ω) $= 0.5 Z_0$.

$$V_i = \frac{Z_i}{Z_g + Z_i} V_g, \qquad I_i = \frac{V_g}{Z_g + Z_i},$$

where $Z_i = Z_0 \dfrac{0.5 Z_0 + jZ_0 \tan\beta l}{Z_0 + j0.5 Z_0 \tan\beta l} = Z_0 \dfrac{1 + j2\tan\beta l}{2 + j\tan\beta l}$.

$$\therefore V_i = \frac{1 + j2\tan\beta l}{3(1 + j\tan\beta l)} V_g = \frac{1}{30}\left(\frac{1 + j2\tan\beta l}{1 + j\tan\beta l}\right) \text{ (V)}.$$

$$I_i = \frac{2 + j\tan\beta l}{3Z_0(1 + j\tan\beta l)} V_g = \frac{2}{3}\left(\frac{2 + j\tan\beta l}{1 + j\tan\beta l}\right) \text{ (mA)}.$$

Setting $Z_g = Z_0$ and $\Gamma_g = 0$ in Eqs. (9-120a) and (9-120b), we have $V_L = V(z'=0) = \dfrac{V_g Z_0}{Z_0 + Z_g} e^{-j\beta l}(1 + \Gamma)$ $\left\{\Gamma = \dfrac{R_L - Z_0}{R_L + Z_0} = -\dfrac{1}{3}\right\}$

$$= \frac{1}{30} e^{-j\beta l} \text{ (V)}.$$

$$I_L = I(z'=0) = \frac{V_g}{Z_0 + Z_g} e^{-j\beta l}(1 - \Gamma) = \frac{4}{3} e^{-j\beta l} \text{ (mA)}.$$

b) $S = \frac{1+|\Gamma|}{1-|\Gamma|} = 2.$

c) $(P_{av})_L = \frac{1}{2}\mathcal{R}e(V_L I_L^*) = \frac{1}{2}\left(\frac{1}{30}\right)\left(\frac{4}{3}\times 10^{-3}\right) = 2.22\times 10^{-5}$ (W) $= 0.0222$ (mW).

If $R_L = 50\ (\Omega) = Z_0$, $V_L = \frac{V_g}{2}e^{-j\beta \ell}$, $I_L = \frac{V_g}{2Z_0}e^{-j\beta \ell}$

$\longrightarrow Max.(P_{av})_L = \frac{V_g^2}{8Z_0} = 2.50\times 10^{-5}$ (W).

P.9-31 $V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$, $I(z) = \frac{1}{R_0}(V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z})$.

$V_{inc.}(z=0) = V_0^+ = \frac{V_g}{2}$, $I_{inc.}(z=0) = I_0^+ = \frac{V_g}{2R_0}$.

a) $P_{inc.} = \frac{1}{2}\mathcal{R}e\left[V_0^+(I_0^+)^*\right] = |V_0^+|^2/2R_0 = V_g^2/8R_0.$

b) $P_L = \frac{1}{2}\mathcal{R}e\left[V(z)I^*(z)\right] = \frac{1}{2R_0}\mathcal{R}e\left[(V_0^+e^{-j\beta z}+V_0^-e^{j\beta z})(V_0^{+*}e^{j\beta z}-V_0^{-*}e^{-j\beta z})\right]$

$= \frac{1}{2R_0}\left\{|V_0^+|^2 - |V_0^-|^2\right\} = \frac{|V_0^+|^2}{2R_0}\left\{1-|\Gamma|^2\right\} = \frac{V_g^2}{8R_0}\left\{1-|\Gamma|^2\right\}.$

c) $\frac{P_L}{P_{inc.}} = 1 - |\Gamma|^2 = 1 - \left(\frac{S-1}{S+1}\right)^2 = \frac{4S}{(S+1)^2}.$

d) $P_{inc.} = \frac{100^2}{8\times 50} = 25$ (W), $\Gamma = \frac{Z_L - R_0}{Z_L + R_0} = \frac{-j25}{100-j25} = 0.243\angle{-76°}$,

$S = \frac{1+|\Gamma|}{1-|\Gamma|} = 1.64$, $P_L = 25(1-0.243^2) = 23.5$ (W).

$P_L = \frac{1}{2}|I_L|^2 R_L \longrightarrow |I_L| = \sqrt{\frac{2P_L}{R_L}} = 0.97$ (A), $|V_L| = |I_L Z_L| = 54.2$ (V).

P.9-32 $\Gamma_g = 0$, $\Gamma = \frac{Z_L - R_0}{Z_L + R_0} = \frac{j-1}{j+1} = j$, $z = \ell - z'$, $\beta = \omega\sqrt{LC}$.

$\ell = \lambda/4$, $e^{-j\beta z} = e^{-j\beta(\lambda/4 - z')} = e^{-j(\pi/2 - \beta z')} = -j e^{j\beta z'}$.

a) From Eq.(9-153a): $V(z') = -j\frac{V_g}{2}e^{j\beta z'}(1+j e^{-j2\beta z'}) = 55(e^{-j\beta z'} - j e^{j\beta z'})$.

From Eq.(9-153b): $I(z') = -j\frac{V_g}{2Z_0}e^{j\beta z'}(1-j e^{-j2\beta z'}) = -1.1(e^{-j\beta z'} + j e^{j\beta z'})$.

b) $v(z',t) = \mathcal{I}m\left[V(z')e^{j\omega t}\right] = 55\left[\sin(\omega t - \beta z') - \cos(\omega t + \beta z')\right]$ (V),

$i(z',t) = -1.1\left[\sin(\omega t - \beta z') + \cos(\omega t + \beta z')\right]$ (A).

c) At the load, $z' = 0$,

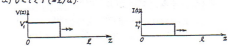
$p_L(t) = v(0,t)\,i(0,t)$
$= 60.5(\cos^2\omega t - \sin^2\omega t) = 60.5\cos(2\omega t)$ (W).

$V_L = \frac{V_g}{2}(1-j)$, $I_L = -\frac{V_g}{2Z_0}(1+j)$.

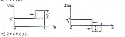
$(P_{av})_L = \frac{1}{2}\mathcal{R}e(V_L I_L^*) = \frac{V_g^2}{4Z_0}\mathcal{R}e(2j) = 0.$

P.9-33 $\Gamma_g = -1, \quad \Gamma_L = +1.$

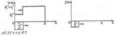
a) $0 < t < T \ (=\ell/u)$.



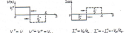
b) $T < t < 2T$.



c) $2T < t < 3T$.



d) $3T < t < 4T$.



$V_1^+ = V_0, \quad V_1^- = V_1^+ = V_0,$ $\qquad I_1^+ = V_0/R_0, \quad I_1^- = -I_1^+ = -V_0/R_0.$

$V_2^+ = -V_1^- = -V_0,$ $\qquad I_2^+ = I_1^- = -V_0/R_0,$

$V_2^- = V_2^+ = -V_0.$ $\qquad I_2^- = -I_2^+ = V_0/R_0.$

At $t = 4T$, both $V(z)$ and $I(z)$ revert back to the conditions at $t=0$, and the cycle repeats itself with a period $4T$.

P9-34 

At the connecting points of two transmission lines with different characteristic impedances Z_{01} and Z_{02}:

$$V_1 = V_2 \longrightarrow V_1^+ + V_1^- = V_2^+$$

$$I_1 = I_2 \longrightarrow I_1^+ + I_1^- = \frac{1}{Z_{01}}(V_1^+ - V_1^-) = \frac{V_2^+}{Z_{02}}$$

Solving $V_1^- = \dfrac{Z_{02}-Z_{01}}{Z_{02}+Z_{01}}V_1^+$, $\quad V_2^+ = \dfrac{2Z_{02}}{Z_{02}+Z_{01}}V_1^+$;

$I_1^- = -\dfrac{Z_{02}-Z_{01}}{Z_{02}+Z_{01}}I_1^+$, $\quad I_2^+ = \dfrac{2Z_{01}}{Z_{02}+Z_{01}}I_1^+$.

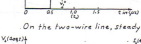
a) $V_1^+ = \dfrac{V_g}{2} = 50\text{ (V)}$, $\quad I_1^+ = \dfrac{V_g}{50+50} = 1\text{ (A)}$;

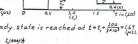
$V_1^- = \dfrac{200-50}{200+50}V_1^+ = 30\text{ (V)}$, $\quad I_1^- = -\dfrac{200-50}{200+50}I_1^+ = -0.6\text{ (A)}$;

$V_2^+ = \dfrac{2\times200}{200+50}V_1^+ = 80\text{ (V)}$, $\quad I_2^+ = \dfrac{2\times50}{200+50}I_1^+ = 0.4\text{ (A)}$.

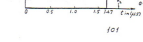
No transient waves on the coaxial cable after V_1^- and I_1^- reach the input terminals (at $2t_1 = 2\ell_1/u_1 = 400/2\times10^8 = 2\,\mu s$), and no transient waves on the two-wire line after V_2^+ and I_2^+ reach the load R_L (at $t_2 = t_1 + \ell_2/u_2 = 1\times10^{-6} + 400/3\times10^8 = 2.33\times10^{-6}\text{ s} = 2.33\,\mu s$).

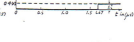
b) On the coaxial cable it takes $t_1/2 = 0.5\,(\mu s)$ for V_1^+ and I_1^+ to reach the midpoint ($x = 100$ m). The reflected waves V_1^- and I_1^- arrive at the midpoint at $t = 3t_1/2 = 1.5\,(\mu s)$. There are no changes after that.





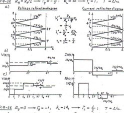
On the two-wire line, steady state is reached at $t = t_1 + \frac{200}{3\times10^8} = 1.67\,(\mu s)$.





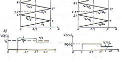
P.9-35 $R_g = R_0/2 \longrightarrow \Gamma_g = -\frac{1}{3}$, $R_L = \infty \longrightarrow \Gamma_L = 1$; $T = \ell/u$

a) Voltage reflection diagram Current reflection diagram

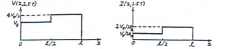


P.9-36 $R_g = 0 \longrightarrow \Gamma_g = -1$, $R_L = 2R_0 \longrightarrow \Gamma_L = \frac{1}{3}$; $T = \ell/u$.

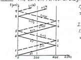
a) Voltage reflection diagram Current reflection diagram



c)



P.9-37 The current reflection diagram for Example 9-11 is:

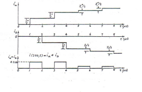


$\Gamma_g = -\frac{1}{3}$, $\Gamma_L = -1$

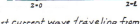
$T = 2\ (\mu s)$.

Indices on the directed lines are normalized with respect to

$$I_1^+ = \frac{V_1^+}{R_0} = \frac{10}{50} = 0.2\ (A).$$



P.9-38 Use the equivalent circuit in Fig. 9-25(d) to study transient voltages and currents:



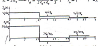
a) Amplitude of first current wave traveling from $z=0$ to $z=\ell$ is $I_1^+ = -\frac{V_0}{2R_0+R_0} = -\frac{V_0}{3R_0}$.

$I_L = -I_1$, $V_L(t) = R\,I_L(t)$.

Refer to Fig. 9-19.

$R = 2R_0 \longrightarrow \Gamma_g = \frac{2R_0-R_0}{2R_0+R_0} = \frac{1}{3}$, $\Gamma_L = 1$ $T = \ell/c$

b)



$R = \frac{R_0}{2} \longrightarrow \Gamma_g = \frac{(1/2)-1}{(1/2)+1} = -\frac{1}{3}$. $\Gamma_L = 1$ $T = \ell/c$.

P.9-39 a) Governing equation at the load for $t \geq T$:

$$L_L \frac{di_L(t)}{dt} + (R_0+R_L)\,i_L(t) = 2V_1^+.$$

Solution: $i(t) = \frac{2V_1^+}{R_0+R_L}\left[1 - e^{-\frac{R_0+R_L}{L_L}(t-T)}\right]$, $t \geq T$

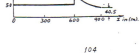
For the present problem, $V_1^+ = V_0/2 = 50\,(V)$, $R_0+R_L = 60\,(\Omega)$,

$L_L = 48\times10^{-6}\,(H)$; $T = \ell/c = 900/3\times10^8 = 3\times10^{-6}\,(s)$.

$i_L(t) = \frac{5}{3}\left[1 - e^{-1.25\times10^6(t-3\times10^{-6})}\right]$, $t \geq 3\times10^{-6}\,(s)$.

$v_L(t) = R_L i_L + L_L\frac{di_L}{dt} = \frac{50}{3}\left[1 + 5e^{-1.25\times10^6(t-3\times10^{-6})}\right]$, $t \geq 3\times10^{-6}$ s

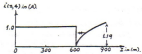
b)



At $t_1 = 4\,(\mu s)$,

$v_L(4) = \frac{50}{3}\left[1 + e^{-1.25(4-3)}\right]$

$= 40.5\,(V)$.



At $t_1 = 4(\mu s)$,

$i_L(4) = \frac{5}{3}\left[1 - e^{-1.25(4-3)}\right]$

$= 1.19$ (A).

<u>P.9-40</u> From Eq.(9-171): $v_L(t) = 2V_1^+ - R_0 i_L(t)$. ①

At the load: $i_L(t) = \frac{v_L(t)}{R_L} + C_L \frac{dv_L(t)}{dt}$ ②

Substituting ② in ① $C_L \frac{dv_L(t)}{dt} + \left(\frac{1}{R_L} + \frac{1}{R_0}\right)v_L(t) = \frac{2}{R_0}V_1^+$. ③

a) Solution of ③: $v_L(t) = 2V_1^+ \frac{R_{ll}}{R_0}\left[1 - e^{-(t-T)/R_{ll}C_L}\right]$, $t \geq T$.

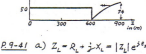
For this problem: $V_1^+ = \frac{V_0}{2} = 50$ (V), $R_{ll} = \frac{R_L R_0}{R_L + R_0}$.

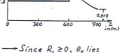
$T = 3(\mu s)$, $R_{ll} = 47.6(\Omega)$, $R_{ll}C_L = 6.66 \times 10^{-7}$ (s).

$v_L(t) = 95.2\left[1 - e^{-(t-3)/0.666}\right]$ (V), t in (μs).

From ②: $i_L(t) = 0.095 + 1.905\, e^{-(t-3)/0.666}$ (A).

b) At $t_1 = 4(\mu s)$, $v_L(4) = 74$ (V), $i_L(4) = 0.519$ (A).





<u>P.9-41</u> a) $Z_L = R_L + jX_L = |Z_L|e^{j\theta_z}$ ⟶ Since $R_L \geq 0$, θ_z lies between $\pm\pi/2$.

$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = |Z_0|e^{j\theta_0}$ ⟶ Since R, L, G, C all ≥ 0, θ_0 lies between $\pm\frac{\pi}{4}$.

$\therefore\ z_L = \frac{Z_L}{Z_0} = \left|\frac{Z_L}{Z_0}\right| e^{j(\theta_z - \theta_0)} = |z_L|e^{j\theta_L}$ ⟶ θ_L must lie between $\pm 3\pi/4$.

b) $|\Gamma|^2 = \left|\frac{z_L - 1}{z_L + 1}\right|^2 = \frac{(|z_L|e^{j\theta_L} - 1)(|z_L|e^{-j\theta_L} - 1)}{(|z_L|e^{j\theta_L} + 1)(|z_L|e^{-j\theta_L} + 1)} = \frac{|z_L|^2 + 1 - 2|z_L|\cos\theta_L}{|z_L|^2 + 1 + 2|z_L|\cos\theta_L} > 1$ when $\cos\theta_L < 0$.

c) $\frac{\partial |\Gamma|^2}{\partial |z_L|} = 0$ ⟶ $|z_L| = 1$; at $\theta_L = 3\pi/4$,

$\text{Max}|\Gamma| = 1 + \sqrt{2} = 2.414$.

P.9-42 $f = 2\times10^8$ (Hz), $\lambda = \frac{c}{f} = 1.5$ (m)

a) Open-circuited line, $\ell = 1$ (m), $\ell/\lambda = 0.667$.
Smith chart: Start from P_{oc} on the extreme right, rotate clockwise one complete revolution ($\Delta z' = \lambda/2$) and continue on for an additional 0.167λ to 0.417λ on the "wavelength toward generator" scale. Read
$x = -j0.575 \longrightarrow Z_i = 75\times(-j0.575) = -j43.1$ (Ω).
Draw a straight line from the $(0-j0.575)$ point through the center and intersect at $(0+j1.74)$ on the opposite side of the chart. $\longrightarrow Y_i = \frac{1}{75}\times(j1.74) = j0.0232$ (S).

b) Short-circuited line, $\ell = 0.8$ (m), $\ell/\lambda = 0.533$.
Start from the extreme left point P_{sc}, rotate clockwise one complete revolution and continue on for an additional 0.033λ to read $x = j0.21 \longrightarrow Z_i = 75\times j0.21 = j15.8$ (Ω).
Draw a straight line from the $(0+j0.21)$ point through the center and intersect at $(0-j4.75)$ on the opposite side of the chart. $\longrightarrow Y_i = \frac{1}{75}\times(-j4.75) = -j0.063$ (S).

P.9-43

$z_L = \frac{1}{50}(30+j10) = 0.6+j0.2$.

a) 1. Locate $z_L = 0.6+j0.2$ on Smith chart (Point P_1).
2. With center at O draw a $|\Gamma|$-circle through P_1, intersecting OP_{oc} at 1.77. $\longrightarrow S = 1.77$.

b) $\Gamma = \frac{1.77-1}{1.77+1}e^{j146°} = 0.28\,e^{j146°}$.

c) 1. Draw line OP_1, intersecting the periphery at P_1'. Read 0.046 on "wavelengths toward generator" scale.
2. Move clockwise by 0.101λ to 0.147 (Point P_2').
3. Join O and P_2', intersecting the $|\Gamma|$-circle at P_2.
4. Read $z_i = 1+j0.59$ at P_2.
$Z_i = 50 z_i = 50+j29.5$ (Ω).

d) Extend line $P_2'P_2O$ to P_3. Read $y_i = 0.75 - j0.43$.

$$Y_i = \frac{1}{50} y_i = 0.015 - j0.009 \ (S).$$

e) There is no voltage minimum on the line, but $V_L < V_i$.

P. 9-44

$$z_L = \frac{1}{50}(30 - j10) = 0.6 - j0.2$$

a) Locate $z_L = 0.6 - j0.2$ on Smith chart (Point P_1). With center at O draw a $|\Gamma|$-circle through P_1, intersecting line OP_{sc} at 1.77. ⟶ $S = 1.77$.

b) $\Gamma = 0.28 e^{j216°}$

c) 1. Draw line OP_1, intersecting the periphery at P_1'. Read 0.454 on "wavelengths toward generator" scale.

2. Move clockwise by 0.101λ to 0.055 (Point P_2').

3. Join O and P_2', intersecting the $|\Gamma|$-circle at P_2.

4. Read $z_i = 0.61 + j0.23$ at P_2.

$$Z_i = 50 z_i = 30.5 + j11.5 \ (\Omega).$$

d) Extend line $P_2'P_2O$ to P_3. Read $y_i = 1.42 - j0.54$.

$$Y_i = \frac{1}{50} y_i = 0.0284 - j0.0108 \ (S).$$

e) There is a voltage minimum at $z_m' = 0.046\lambda$.

P. 9-45 $\lambda/2 = 25$, $\lambda = 50$ (cm).

First voltage minimum occurs at $z_m' = \frac{5}{50} = 0.1\lambda$.

a) 1. Start from P_{sc} and rotate counterclockwise 0.10λ toward the load to P_1'.

2. Draw the $|\Gamma|$-circle, intersecting line OP_{sc} at 2 ($S = 2$).

3. Join OP_1', intersecting the $|\Gamma|$-circle at P_1.

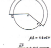
4. Read $z_L = 0.675 - j0.475$.

$$Z_L = 50 z_L = 33.75 - j23.75 \ (\Omega).$$

b) $\Gamma = \frac{2-1}{2+1} e^{j\theta_\Gamma} = \frac{1}{3} e^{j252.5°}$.

c) If $Z_L = 0$, the first voltage minimum would be at $z_m' = \lambda/2 = 25$ (cm) from the short-circuit.

P9-46

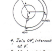


a) $z_i = \frac{1}{100}(40 - j280) = 0.40 - j2.80$.

1. Enter z_i on Smith chart (Point P).
2. Join O and P, and extend to P'.
3. Read on "wavelengths toward generator" scale: 0.303.

$$\beta\ell = 0.606\pi, \quad \ell = 1.5 \text{ (m)} \longrightarrow \beta = 1.269 \text{ (rad/m)}.$$

$$\frac{\overline{OP}}{\overline{OP'}} = 0.915 \longrightarrow \alpha = \frac{1}{2\ell} \ln \frac{1}{0.915} = 0.0297 \text{ (Np/m)}.$$



b) 1. Enter $z_L = 0.5 + j0.5$ on Smith chart (Point P_1).
2. Draw line from O through P_1 to P_1'. Read on "wavelengths toward generator" scale: 0.088.
3. Move clockwise by 0.303λ to 0.391 (Point P'').
4. Join OP'', intersecting the $|\Gamma|$-circle through P_1 at P'.
5. Mark point P on line OP' such that $\frac{\overline{OP}}{\overline{OP'}} = 0.915$.
6. Read at P: $z_i = 0.625 - j0.59 \longrightarrow Z_i = 62.5 - j59.0 \ (\Omega)$

c) 1. Move clockwise from P_m on "wavelengths toward generator" scale to 0.15, say P'.

2. Join OP'.

3. Mark point P on line OP' such that
$$\overline{OP} = e^{-2\alpha\ell}\,\overline{OP'} = 0.967\,\overline{OP'}.$$

4. Read at P: $z_i = 0.065 + j1.38 \longrightarrow Z_i = 6.5 + j138\ (\Omega)$.

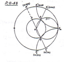
P.9-47 $f = 2\times10^8$ (Hz), $\lambda = 1.5$ (m) $\longrightarrow \ell = \frac{\lambda}{4} = 0.375$ (m).

$$Z_0 = \sqrt{73\times300} = 148\ (\Omega).$$

For two-wire transmission line: $Z_0 = 120\cosh^{-1}\left(\frac{D}{2a}\right)$.

$D = 2$ (cm) $\longrightarrow a = 0.56$ (cm).

P.9-48



$z_L = 0.5 + j0.5$.

$y_L = 1 - j$.

a) See construction.

P_1: $z_L = 0.5 + j0.5$.

P_2: $y_L = 1 - j1 = y_1 \longrightarrow d_1 = 0$.

P_3: $y_2 = 1 + j1$.

$\longrightarrow d_2 = 0.162\lambda + (0.5 - 0.338)\lambda = 0.324\lambda$.

P_2'': $b_1 = j1 \longrightarrow \ell_1 = (0.5 + 0.125)\lambda = 0.375\lambda$.

P_3'': $b_2 = -j1 \longrightarrow \ell_2 = (0.375 - 0.25)\lambda = 0.125\lambda$.

b) For $Z_0' = 75 = 1.5\,Z_0$, $Y_0' = 0.667\,Y_0$.

The required normalized stub admittances are $b_2' = -b_1' = \frac{j}{0.667} = j1.5$.

	$(Z_0)_{stub} = (Z_0)_{line}$	$(Z_0)_{stub} = 1.5(Z_0)_{line}$
$z_L = 0.5 + j0.5$, $y_L = 1 - j1$	$d_1 = 0$, $\ell_1 = 0.375\lambda$	$d_1' = 0$, $\ell_1' = 0.406\lambda$
	$d_2 = 0.324\lambda$, $\ell_2 = 0.125\lambda$	$d_2' = 0.324\lambda$, $\ell_2' = 0.094\lambda$

P.9-49 $z_L = 0.5 + j0.5$

Use Smith chart as an impedance chart. Same construction as that in problem P.9-34 except P_{sc} would be on the extreme left (marked by a x), and g=1 circle becomes r=1 circle.

P_1: $z_L = 0.5 + j0.5$; P_2: $z_{i2} = 1 + j1$ with $d_1 = (0.162 - 0.088)\lambda = 0.074\lambda$.

P_3: $z_{i3} = 1 - j1$ with $d_2 = (0.338 - 0.088)\lambda = 0.250\lambda$.

To achieve a match with a series stub having $R_0' = \frac{35}{50} R_0$, we need a normalized stub susceptance $-j\frac{50}{35} = -j1.43$ for solution corresponding to P_2. From Smith chart we obtain the required stub length $\ell_2 = 0.347\lambda$.

Similarly for solution corresponding to P_3, a stub with a normalized susceptance $+j1.43$ is needed, which requires a stub length $\ell_3 = 0.153\lambda$.

P.9-50

$z_L = 0.33 + j0.33$.

P_L: $y_L = 1.50 - j1.50$ (0.206λ at P_L'),

P_{A1}: $y_{A1} = 1.50 - j1.80$ (0.204λ at P_{A1}'),

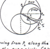
P_{A2}: $y_{A2} = 1.50 + j0.14$ (0.269λ at P_{A2}'),

P_{B1}: $y_{B1} = 1.00 + j1.60$ (0.179λ at P_{B1}'),

P_{B2}: $y_{B2} = 1.00 + j0.40$ (0.146λ at P_{B2}').

	a) Short-circuited stubs	b) Open-circuited stubs
$(y_{sA})_1 = y_{A1} - y_L = -j0.30$	$\ell_{A1} = 0.203\lambda$	$\ell_{A1} = 0.453\lambda$
$(y_{sA})_2 = y_{A2} - y_L = j1.36$	$\ell_{A2} = 0.399\lambda$	$\ell_{A2} = 0.149\lambda$
$(y_{sB})_1 = -j1.60$	$\ell_{B1} = 0.089\lambda$	$\ell_{B1} = 0.339\lambda$
$(y_{sB})_2 = -j0.40$	$\ell_{B2} = 0.189\lambda$	$\ell_{B2} = 0.439\lambda$

P.9-51



$Y_L = \frac{300}{100+j50} = 2.4 - j1.2.$

Point P_L on Smith chart. $(0.280\lambda$ at $P_L')$

Since the rotated $g=1.0$ circle is tangent to the $g=2.0$ circle, an added line length d_L is needed to convert g_L (2.4) to 2.0, moving from P_L along the $|\Gamma|$-circle to P_{L1} (not shown) on the $g=2.0$ circle $(0.291\lambda$ at $P_{L1}')$. Note that P_{L1} is different from P_A, the point of tangency between the $g=2.0$ and rotated $g=1.0$ circles.

a) Min. $d_L = 0.291\lambda - 0.280\lambda = 0.011\lambda$.

b) P_A : $y_A = 2 - j1$ $(0.287\lambda$ at $P_A')$.

P_B : $y_B = 1 + j1$ $(0.162\lambda$ at $P_B')$.

$Y_{SA} = Y_A - Y_{L1} = (2-j1) - (2-j1.35) = j0.35 \longrightarrow \ell_A = 0.304\lambda$.

$Y_{SB} = -j1 \longrightarrow \ell_B = 0.125\lambda$.

P.9-52 Let $\theta = \beta d_0 = \frac{2\pi}{\lambda} d_0$.

Require: $g_L \leq \frac{1}{\sin^2\theta}$ (Analytical solution†)

d_0	θ	No solution if g_L is
$\lambda/16$	22.5°	>6.83
$\lambda/8$	45°	>2.0
$\lambda/4$	90°	>1.0
$3\lambda/8$	135°	>2.0
$7\lambda/16$	157.5°	>6.83

† See D.K. Cheng and C.H. Liang, "Computer Solution of Double-Stub Impedance-Matching Problems," IEEE Transactions on Education, vol. E-25, pp. 120-123, November 1982.

Chapter 10

Waveguides and Cavity Resonators

P.10-1

$$\nabla \times \bar{E} = -j\omega\mu\bar{H}. \qquad ①$$

$$\nabla \times \bar{H} = j\omega\epsilon\bar{E}. \qquad ②$$

From ①, $(\bar{\nabla}_T + \bar{a}_z\frac{\partial}{\partial z}) \times (\bar{E}_T + \bar{a}_z E_z) = -j\omega\mu(\bar{H}_T + \bar{a}_z H_z)$.

$$\longrightarrow \bar{\nabla}_T \times (\bar{a}_z E_z) + \gamma\bar{E}_T \times \bar{a}_z = -j\omega\mu\bar{H}_T$$

$$\longrightarrow \bar{\nabla}_T E_z \times \bar{a}_z + \gamma\bar{E}_T \times \bar{a}_z = -j\omega\mu\bar{H}_T \qquad ③$$

$$(\because \bar{\nabla}_T \times (\bar{a}_z E_z) = \bar{\nabla}_T E_z \times \bar{a}_z)$$

Similarly from ② we obtain

$$\bar{\nabla}_T H_z \times \bar{a}_z + \gamma\bar{H}_T \times \bar{a}_z = j\omega\epsilon\bar{E}_T. \qquad ④$$

Combining ③ and ④, we have

$$\omega^2\mu\epsilon\bar{E}_T = \gamma[\bar{\nabla}_T E_z \times \bar{a}_z + \gamma\bar{E}_T \times \bar{a}_z] \times \bar{a}_z - j\omega\mu\bar{\nabla}_T H_z \times \bar{a}_z.$$

$$\longrightarrow \bar{E}_T = -\frac{1}{h^2}(\gamma\bar{\nabla}_T E_z - \bar{a}_z j\omega\mu \times \bar{\nabla}_T H_z), \quad h^2 = \gamma^2 + \omega^2\mu\epsilon.$$

Similarly, $\bar{H}_T = -\frac{1}{h^2}(\gamma\bar{\nabla}_T H_z + \bar{a}_z j\omega\epsilon \times \bar{\nabla}_T E_z)$.

P.10-2 a)

From Eq. (10-38):

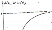
$$\left(\frac{\beta}{k}\right)^2 + \left(\frac{f_c}{f}\right)^2 = 1.$$

From Eq. (10-37):

$$\left(\frac{u_g}{u}\right)^2 + \left(\frac{f_c}{f}\right)^2 = 1.$$

Both are equations of a unit circle.

b)

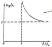


From Eq. (10-38):

$$\left(\frac{\beta}{k}\right)^2 = 1 - \frac{1}{(f/f_c)^2}.$$

From Eq. (10-42):

$$\left(\frac{u}{u_p}\right)^2 = 1 - \frac{1}{(f/f_c)^2}.$$



From Eq. (10-39):

$$\left(\frac{\lambda_g}{\lambda}\right)^2 = \frac{(f/f_c)^2}{(f/f_c)^2 - 1}.$$

c) At $f/f_c = 1.25$:

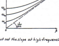
$u_p/u = 1.67$,

$u_g/u = 0.60$,

$\beta/k = 0.60$,

$\lambda_g/\lambda = 1.67$.

P.10-3 a) For parallel-plate waveguides:



$$\omega^2\mu\epsilon - \beta^2 = \left(\frac{n\pi}{b}\right)^2.$$

$$\omega_{c1} = \frac{\pi}{b\sqrt{\mu\epsilon}},$$

$$\omega_{c2} = \frac{2\pi}{b\sqrt{\mu\epsilon}},$$

$$\omega_{c3} = \frac{3\pi}{b\sqrt{\mu\epsilon}}.$$

a) Constitutive parameters ϵ and μ affect both ω_c and the slope of the ω-β curves; b affects ω_c but not the slope at high frequencies. b) Yes.

P.10-4 Field expressions for TM_n modes, from Eqs. (10-63, 64 & 65):

$$E_z^0(y) = A_n \sin(n\pi y/b),$$

$$H_x^0(y) = \frac{j\omega\epsilon}{h} A_n \cos(n\pi y/b),$$

$$E_y^0(y) = -\frac{\gamma}{h} A_n \cos(n\pi y/b).$$

Surface charge densities:

$$\rho_{sl} = \bar{a}_n \cdot \bar{D}\Big|_{y=0} = \epsilon E_y^0(0) = -\frac{\gamma\epsilon}{h} A_n,$$

$$\rho_{su} = \bar{a}_n \cdot \bar{D}\Big|_{y=b} = -\epsilon E^0(b) = (-1)^n \frac{\gamma\epsilon}{h} A_n.$$

Surface current densities:

$$\bar{J}_{sl} = \bar{a}_n \times \bar{H}\Big|_{y=0} = \bar{a}_y \times \bar{H}(0) = -\bar{a}_z \frac{j\omega\epsilon}{h} A_n,$$

$$\bar{J}_{su} = \bar{a}_n \times \bar{H}\Big|_{y=b} = -\bar{a}_y \times \bar{H}(b) = \bar{a}_z (-1)^n \frac{j\omega\epsilon}{h} A_n = \begin{cases} \bar{J}_{sl} & \text{for } n \text{ odd}, \\ -\bar{J}_{sl} & \text{for } n \text{ even}. \end{cases}$$

P. 10-5 Field expressions for TE_n modes, from Eqs. (10-83, 84 & 85):

$$H_z^0(y) = B_n \cos(n\pi y/b),$$
$$H_y^0(y) = \frac{\gamma}{h} B_n \sin(n\pi y/b),$$
$$E_x^0(y) = \frac{j\omega\mu}{h} B_n \sin(n\pi y/b).$$
$$\bar{J}_{sl} = \bar{a}_y \times \bar{H}(0) = \bar{a}_x B_n ,$$
$$\bar{J}_{su} = -\bar{a}_y \times \bar{H}(b) = \bar{a}_x (-1)^{n+1} B_n = \begin{cases} \bar{J}_{sl} & \text{for } n \text{ odd}, \\ -\bar{J}_{sl} & \text{for } n \text{ even}. \end{cases}$$

P. 10-6 a) Set $n=2$ in the field expressions in problem P. 10-4.

—— Electric field lines
× • Magnectic field lines
Equation for electric field lines: $\cos\beta z = \frac{\cos(2\pi y_0/b)}{\cos(2\pi y/b)}$.

b) Set $n=2$ in the field expressions in problem P. 10-5.

× • Electric field lines
—— Magnetic field lines
Equation for magnetic field lines: $\cos\beta z = \frac{\sin(2\pi y_0/b)}{\sin(2\pi y/b)}$.

P. 10-7 Using the field expressions in problem P. 10-5 above:

$$\bar{\mathcal{P}}_{av} = \frac{1}{2}\mathcal{R}e(\bar{E} \times \bar{H}^*) = \frac{1}{2}\mathcal{R}e(\bar{a}_z E_x^0 H_y^{0*} - \bar{a}_y E_x^0 H_z^{0*}).$$
$$\bar{\mathcal{P}}_{av} \cdot \bar{a}_z = \frac{1}{2}\mathcal{R}e(E_x^0 H_y^{0*}) = \frac{\omega\mu\beta}{2h^2} B_n^2 \sin^2\left(\frac{n\pi y}{b}\right).$$
$$(P_z)_{av} = \int_0^b \bar{\mathcal{P}}_{av} \cdot \bar{a}_z \, dy = \frac{\omega\mu\beta b}{4h^2} B_n^2 \quad \text{(per unit guide width)}.$$
$$(w_e)_{av} = \frac{\epsilon}{4}\mathcal{R}e(\bar{E}^0 \cdot \bar{E}^{0*}) = \frac{\epsilon\omega^2\mu^2}{4h^2} B_n^2 \sin^2\left(\frac{n\pi y}{b}\right).$$
$$(W_e)'_{av} = \int_0^b (w_e)_{av} dy = \frac{\epsilon\omega^2\mu^2 b}{8h^2} B_n^2 = (W_m)'_{av} \quad \text{(per unit guide width)}$$
From Eq. (10-92): $$u_{en} = \frac{(P_z)_{av}}{(W_e)'_{av} + (W_m)'_{av}} = \frac{\omega\beta}{k^2} = u\sqrt{1-\left(\frac{f_c}{f}\right)^2},$$
which is the same as Eq. (10-104).

P.10-8 Given: $\sigma_c = 5.80\times10^{7}$ (S/m), $\epsilon_r = 2.25$, $\mu_r = 1$,
$\sigma = 10^{-10}$ (S/m), $b = 5\times10^{-2}$ (m), $f = 10^{10}$ (Hz).

a) TEM mode

$$\beta = \omega\sqrt{\mu\epsilon} = 314.2 \text{ (rad/m)},$$
$$\alpha_d = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} = 1.257\times10^{-8} \text{ (Np/m)},$$
$$\alpha_c = \frac{1}{b}\sqrt{\frac{\pi f\epsilon}{\sigma_c}} = 2.076\times10^{-3} \text{ (Np/m)},$$
$$u_p = u_g = u = \frac{1}{\sqrt{\mu\epsilon}} = 2\times10^{8} \text{ (m/s)},$$
$$\lambda_g = \lambda = \frac{u}{f} = 2\times10^{-2} \text{ (m)}.$$

b) TM_1 mode — $(f_c)_{TM_1} = \frac{1}{2b\sqrt{\mu\epsilon}} = 2\times10^{9}$ (Hz) $< f$.

$$F_1 = \sqrt{1-(f_c/f)^2} = 0.9798.$$
$$\beta = \omega\sqrt{\mu\epsilon}\cdot F_1 = 307.8 \text{ (rad/m)},$$
$$\alpha_d = \frac{\sigma\eta}{2F_1} = 1.283\times10^{-8} \text{ (Np/m)},$$
$$\alpha_c = \frac{2R_s}{\eta b F_1} = \frac{2}{bF_1}\sqrt{\frac{\pi f\epsilon}{\sigma_c}} = 4.238\times10^{-3} \text{ (Np/m)},$$
$$u_p = u/F_1 = 2.041\times10^{8} \text{ (m/s)},$$
$$u_g = u\cdot F_1 = 1.960\times10^{8} \text{ (m/s)},$$
$$\lambda_g = \lambda/F_1 = 2.041\times10^{-2} \text{ (m)}.$$

c) TM_2 mode — $(f_c)_{TM_2} = \frac{1}{b\sqrt{\mu\epsilon}} = 4\times10^{9}$ (Hz) $< f$.

$$F_2 = \sqrt{1-(f_c/f)^2} = 0.9165.$$
$$\beta = \omega\sqrt{\mu\epsilon}\cdot F_2 = 287.9 \text{ (rad/m)},$$
$$\alpha_d = \frac{\sigma\eta}{2F_2} = 1.371\times10^{-8} \text{ (Np/m)},$$
$$\alpha_c = \frac{2}{bF_2}\sqrt{\frac{\pi f\epsilon}{\sigma_c}} = 4.530\times10^{-3} \text{ (Np/m)},$$
$$u_p = u/F_2 = 2.182\times10^{8} \text{ (m/s)},$$
$$u_g = u\cdot F_2 = 1.833\times10^{8} \text{ (m/s)},$$
$$\lambda_g = \lambda/F_2 = 2.182\times10^{-2} \text{ (m)}.$$

P.10-9 a) TE_1 mode — $(f_c)_{TE_1} = (f_c)_{TM_1} = 2\times10^{9}$ (Hz) $< f$.

All required quantities are the same as those for the TM_1 mode in problem P.10-8 (b), except α_c. Using Eq. (10-83), we have

$$\alpha_c = \frac{2}{bF_1}\sqrt{\frac{\pi f\epsilon}{\sigma_c}}\left(\frac{f_c}{f}\right)^2 = 1.695\times10^{-4} \text{ (Np/m)}.$$

b) TE$_2$ mode —— $(f_c)_{TE_2} = (f_c)_{TM_2} = 4\times10^9$ (Hz) $< f$.

All required quantities are the same as those for the TM$_2$ mode in problem P.10-8 (c), except α_c.

$$\alpha_c = \frac{2}{b f_2}\sqrt{\frac{\pi f \epsilon}{\sigma_c}}\left(\frac{f_c}{f}\right)^2 = 7.249\times10^{-4} \text{ (Np/m)}.$$

P.10-10 For TM$_n$ modes in a parallel-plate waveguide,

$$\alpha_c = \frac{2}{\eta b}\sqrt{\frac{\pi\mu_c f_c}{\sigma_c}}\frac{1}{\sqrt{(f_c/f)[1-(f_c/f)^2]}}$$

$$= \frac{2}{\eta b}\sqrt{\frac{\pi\mu_c f_c}{\sigma_c}}\frac{1}{\sqrt{F(x)}},$$

where $F(x) = x - x^3$, $x = f_c/f$.

a) To find minimum α_c, set

$$\frac{\partial F(x)}{\partial x} = 1 - 3x^2 = 0 \longrightarrow x = \frac{1}{\sqrt{3}}.$$

$$\therefore f = \sqrt{3} f_c.$$

b) At $f_c/f = 1/\sqrt{3}$, $\frac{1}{\sqrt{F(x)}} = 1.612$,

and $$\text{min. } \alpha_c = \frac{3.224}{\eta b}\sqrt{\frac{\pi\mu_c f_c}{\sigma_c}}.$$

c) For $\sigma_c = 5.80\times10^7$ (S/m), $b = 5\times10^{-2}$ (m), $\eta = 120\pi$ (Ω), and $\mu_c = 4\pi\times10^{-7}$ (H/m),

$$(f_c)_{TM_1} = \frac{1}{2b\sqrt{\mu_0\epsilon_0}} = 3\times10^9 \text{ (Hz)},$$

$$\text{min. } \alpha_c = 2.444\times10^{-3} \text{ (Np/m)}.$$

P.10-11 Parallel-plate waveguide: $b = 0.03$ (m), $f = 10^{10}$ (Hz).

a) TEM mode

From Eqs. (9-1a) and (9-1b):

$$\begin{cases} E_y^0 = E_0, \\ H_x^0 = -\frac{E_0}{\eta_0}. \end{cases}$$

$$P_{av} = \frac{w}{2}\int_0^b -E_y^0 H_x^0\, dy = \frac{wb}{2\eta_0}E_0^2.$$

Dielectric strength of air: Max. $E_0 = 3\times10^6$ (V/m).

$$\text{Max.}\left(\frac{P_{av}}{w}\right) = \frac{b}{2\eta_0}(3\times10^6)^2 = 358\times10^6 \text{ (W/m)} = 358 \text{ (MW/m)}.$$

b) TM₁ mode

From Eqs. (10-64) and (10-65):

$$\begin{cases} E_y^0(y) = E_0 \cos\left(\frac{\pi y}{b}\right). \\ H_x^0(y) = -\frac{E_0}{\eta_0\sqrt{1-(f_c/f)^2}} \cos\left(\frac{\pi y}{b}\right). \end{cases}$$

$$f_c = \frac{1}{2b\sqrt{\mu_0\epsilon_0}} = 5\times10^9 \text{ (Hz)}.$$

$$P_{av} = \frac{w}{2}\int_0^b -E_y^0(y)H_x^0(y)\,dy = \frac{wbE_0^2}{4\eta_0\sqrt{1-(f_c/f)^2}}.$$

$$\text{Max.}\left(\frac{P_{av}}{w}\right) = \frac{b(3\times10^6)^2}{4\eta_0\sqrt{1-(f_c/f)^2}} = 2.07\times10^8 \text{ (W/m)} = 207 \text{ (MW/m)}$$

c) TE₁ mode

From Eqs. (10-84) and (10-85):

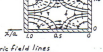
$$\begin{cases} E_x^0(y) = E_0 \sin\left(\frac{\pi y}{b}\right). \\ H_y^0(y) = \frac{E_0}{\eta_0}\sqrt{1-\left(\frac{f_c}{f}\right)^2} \sin\left(\frac{\pi y}{b}\right). \end{cases}$$

$$P_{av} = \frac{w}{2}\int_0^b E_x^0(y)H_y^0(y)\,dy = \frac{wbE_0^2}{4\eta_0}\sqrt{1-(f_c/f)^2}.$$

$$\text{Max.}\left(\frac{P_{av}}{w}\right) = \frac{b(3\times10^6)^2}{4\eta_0}\sqrt{1-(f_c/f)^2} = 1.55\times10^8 \text{ (W/m)} = 155 \text{ (MW/m)}.$$

P.10-12 a) TM_{11} mode

b) TE_{11} mode



—— Electric field lines
---- Magnetic field lines

P.10-13 Equations (10-134) through (10-137) for TM_{11} mode:

$$E_x^0(x,y) = \frac{-j\beta_{11}}{h^2}\left(\frac{\pi}{a}\right)E_0 \cos\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right).$$

$$E_y^0(x,y) = \frac{-j\beta_{11}}{h^2}\left(\frac{\pi}{b}\right)E_0 \sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right).$$

$$E_z^0(x,y) = E_0 \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right).$$

$$H_x^0(x,y) = \frac{j\omega\epsilon}{h^2}\left(\frac{\pi}{b}\right)E_0 \sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right).$$

$$H_y^0(x,y) = \frac{-j\omega\epsilon}{h^2}\left(\frac{\pi}{a}\right)E_0 \cos\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right).$$

a) Surface current densities:

$$\bar{J}_s(y=0) = \bar{a}_n \times \bar{H}\Big|_{y=0} = \bar{a}_y \times [\bar{a}_x H_x^0(x,0) + \bar{a}_y H_y^0(x,0)]$$
$$= -\bar{a}_z H_x^0(x,0) = -\bar{a}_z \frac{j\omega\epsilon}{h^2}\left(\frac{\pi}{b}\right)E_0 \sin\left(\frac{\pi x}{a}\right)e^{-j\beta_{11}z}$$
$$= \bar{J}_s(y=b).$$

$$\bar{J}_s(x=0) = \bar{a}_n \times \bar{H}\Big|_{x=0} = \bar{a}_x \times [\bar{a}_x H_x^0(0,y) + \bar{a}_y H_y^0(0,y)]$$
$$= \bar{a}_z H_y^0(0,y) = -\bar{a}_z \frac{j\omega\epsilon}{h^2}\left(\frac{\pi}{a}\right)E_0 \sin\left(\frac{\pi y}{b}\right)e^{-j\beta_{11}z}$$
$$= \bar{J}_s(x=a).$$

$\bar{J}_s$ at $y=b$

$\bar{J}_s$ at $x=0$.

P.10-14 Rectangular waveguide: $a = 7.21$ (cm), $b = 3.40$ (cm).

Eq. (10-140): $(\lambda_c)_{mn} = \dfrac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$.

Modes with the shortest $\lambda_c < 5$ (cm) are:

Mode	TE_{10}	TE_{20}	TE_{01}	TE_{11}/TM_{11}
λ (cm)	14.4	7.20	6.80	6.15

a) For $\lambda = 10$ (cm), the only propagating mode is TE_{10}.

b) For $\lambda = 5$ (cm), the propagating modes are: TE_{10}, TE_{20}, TE_{01}, TE_{11}, and TM_{11}.

P.10-15 $u_{en} = u\sqrt{1-\left(\frac{f_c}{f}\right)^2}$.

For the TE_{10} mode, $f_{10} = \frac{u}{2a}$.

$$\therefore \quad u_{en} = u\sqrt{1-\left(\frac{u}{2af}\right)^2} = \frac{1}{\sqrt{\mu\epsilon}}\sqrt{1-\frac{1}{\mu\epsilon(2af)^2}}.$$

P.10-16 $(f_c)_{mn} = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = \frac{1}{2a\sqrt{\mu\epsilon}}F(m,n)$.

a) $a=2b$, $F(m,n)=\sqrt{m^2+4n^2}$

Modes	$F(m,n)$
TE_{10}	1
TE_{01}, TE_{20}	2
TE_{11}, TM_{11}	$\sqrt{5}$
TE_{02}	4
TM_{12}	$\sqrt{17}$
TM_{21}	$\sqrt{20}$

b) $a=b$, $F(m,n)=\sqrt{m^2+n^2}$.

Modes	$F(m,n)$
TE_{10}, TE_{01}	1
TE_{11}, TM_{11}	$\sqrt{2}$
TE_{02}, TE_{20}	2
TM_{12}	$\sqrt{5}$
TM_{22}	$2\sqrt{2}$

P.10-17 $f = 3\times10^9$ (Hz), $\lambda = c/f = 0.1$ (m).

Let $a = kb$, $1<k<2$. $(f_c)_{mn} = \frac{3\times10^8}{2a}\sqrt{m^2+k^2n^2}$.

a) $(f_c)_{10} = \frac{1.5\times10^8}{a}$ for the dominant TE_{10} mode.

For $f > 1.2\,(f_c)_{10}$: $a > 0.06$ (m).

The next higher-order mode is TE_{01} with $(f_c)_{01} = \frac{1.5\times10^8}{b}$.

For $f < 0.8\,(f_c)_{01}$: $b < 0.04$ (m).

We choose $a = 6.5$ (cm) and $b = 3.5$ (cm).

b) $u_p = \frac{c}{\sqrt{1-(\lambda/2a)^2}} = 4.70\times10^8$ (m/s).

$\lambda_g = \frac{\lambda}{\sqrt{1-(\lambda/2a)^2}} = 0.157$ (m) $= 15.7$ (cm).

$\beta = \frac{2\pi}{\lambda_g} = 40.1$ (rad/m).

$(Z_{TE})_{10} = \frac{\eta_0}{\sqrt{1-(\lambda/2a)^2}} = 590\ (\Omega)$.

P.10-18 Given: $a = 2.5\times10^{-2}$ (m), $b = 1.5\times10^{-2}$ (m), $f = 7.5\times10^{9}$ (Hz).

a) $\lambda = \frac{c}{f} = \frac{3\times10^8}{7.5\times10^9} = 0.04$ (m),

$F_1 = \sqrt{1-(\lambda/2a)^2} = 0.60,$

$\lambda_g = \lambda/F_1 = 0.0667$ (m) $= 6.67$ (cm),

$\beta = 2\pi/\lambda_g = 94.2$ (rad/m),

$u_p = c/F_1 = 5\times10^8$ (m/s),

$u_g = c\cdot F_1 = 1.8\times10^8$ (m/s),

$(Z_{TE})_{10} = \eta_0/F_1 = 200\pi = 628$ (Ω).

b) $\lambda' = \frac{u}{f} = \frac{\lambda}{\sqrt{2}} = 0.0283$ (m),

$F_2 = \sqrt{1-(\lambda'/2a)^2} = 0.825,$

$\lambda_g' = \lambda'/F_2 = 0.0343$ (m) $= 3.43$ (cm),

$\beta' = 2\pi/\lambda_g' = 183.2$ (rad/m),

$u_p' = u/F_2 = 2.57\times10^8$ (m/s),

$u_g' = u\cdot F_2 = 1.75\times10^8$ (m/s),

$(Z_{TE})_{10} = \frac{\eta_0}{\sqrt{2}\,F_2} = 323$ (Ω).

P.10-19 Given: $a = 7.20\times10^{-2}$ (m), $b = 3.40\times10^{-2}$ (m), $f = 3\times10^{9}$ (Hz).

a) $\lambda_c = 2a = 14.40\times10^{-2}$ (m),

$f_c = \frac{c}{\lambda_c} = 2.08\times10^{9}$ (Hz).

b) $\lambda = \frac{c}{f} = 0.1$ (m), $\sqrt{1-\left(\frac{\lambda}{2a}\right)^2} = 0.720,$

$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{\lambda}{2a}\right)^2}} = 0.139$ (m).

c) $R_s = \sqrt{\frac{\pi f \mu}{\sigma}} = 1.429\times10^{-2}$ (Ω) $\longrightarrow (\alpha_c)_{TE_{10}} = \frac{R_s\left[1+\frac{2b}{a}\left(\frac{f_c}{f}\right)^2\right]}{\eta_0 b\sqrt{1-(f_c/f)^2}} = 2.26\times10^{-3}$ (Np/m).

d) $e^{-\alpha_c z} = \frac{1}{2} \longrightarrow z = \frac{1}{\alpha_c}\ln 2 = 307$ (m).

P.10-20 Given: $a = 2.25\times10^{-2}$ (m), $b = 1.00\times10^{-2}$ (m), $f = 10^{10}$ (Hz).

a) $\lambda = \frac{c}{f} = 3\times10^{-2}$ (m), $\lambda_c = 2a = 4.50\times10^{-2}$ (m).

$\sqrt{1-(f_c/f)^2} = \sqrt{1-(\lambda/\lambda_c)^2} = 0.745.$

Eq. (10-187): $(\alpha_c)_{TE_{10}} = \frac{1}{\eta_0 b}\sqrt{\frac{\pi f \mu_0}{\sigma_c[1-(f_c/f)^2]}}\left[1+\frac{2b}{a}\left(\frac{f_c}{f}\right)^2\right]$

$= 1.295\times10^{-2}$ (Np/m).

b) From Eqs. (10-159), (10-160), and (10-158):

$$E_y^0 = E_0 \sin\left(\frac{\pi x}{a}\right).$$

$$H_x^0 = -\frac{E_0}{\eta_0}\sqrt{1-\left(\frac{f_c}{f}\right)^2}\sin\left(\frac{\pi x}{a}\right).$$

$$H_z^0 = j\left(\frac{f_c}{f}\right)\frac{E_0}{\eta_0}\cos\left(\frac{\pi x}{a}\right).$$

$$P_{av} = \frac{1}{2}\int_0^b\int_0^a(-E_y^0 H_x^0)\,dx\,dy = \frac{E_0^2 ab}{4\eta_0}\sqrt{1-\left(\frac{f_c}{f}\right)^2}.$$

For $P_{av} = 10^3$ (W) at the load (antenna), assuming under matched conditions:

$|E_y^0| = E_0 = 94{,}800$ (V/m), $|H_x^0| = 187.4$ (A/m), $|H_z^0| = 167.6$ (A/m).

The waveguide is 1(m) long. —— The field intensities are higher at the sending end by a factor of $e^{\alpha_c \cdot 1} = 1.138$.

$\therefore$ Max. $|E_y^0| = 10{,}788$ (V/m).

Max. $|H_x^0| = 213.3$ (A/m).

Max. $|H_z^0| = 190.7$ (A/m).

c) $\bar{J}(x=0) = \bar{a}_x \times (\bar{a}_x H_x^0 + \bar{a}_z H_z^0)\big|_{x=0} = -\bar{a}_y H_z^0(0,y) = -\bar{a}_y j\left(\frac{f_c}{f}\right)\frac{E^0}{\eta_0}$

$|\bar{J}(x=0)| = |H_z^0| = 167.6$ (A/m).

$\bar{J}(y=0) = \bar{a}_y \times (\bar{a}_x H_x^0 + \bar{a}_z H_z^0)\big|_{y=0} = -\bar{a}_z H_x^0(x,0) + \bar{a}_x H_z^0(x,0).$

$|\bar{J}(y=0)| = \left[(H_x^0)^2 + (H_z^0)^2\right]^{1/2} = \frac{E_0}{\eta_0}\left\{\left(\frac{f_c}{f}\right)^2 + \left[1 - 2\left(\frac{f_c}{f}\right)^2\sin^2\left(\frac{\pi x}{a}\right)\right]\right\}^{1/2},$

which is maximum at $x = a/2$.

At the sending end: Max $|\bar{J}| = \frac{E_0}{\eta_0}\sqrt{1-\left(\frac{f_c}{f}\right)^2} \times 1.138 = 213.3$ (A/m).

d) Total amount of average power dissipated in 1 (m) of waveguide:

$$P_d = 1000\,(e^{2\alpha_c} - 1) = 1000\,(e^{0.0259} - 1) = 26.2 \text{ (W)}.$$

P.10-21 From problem P.10-20, we have

$$P_{av} = \frac{E_0^2 ab}{4\eta_0}\sqrt{1-\left(\frac{f_c}{f}\right)^2}, \quad \sqrt{1-\left(\frac{f_c}{f}\right)^2} = 0.745.$$

$$\therefore \text{Max. } P_{av} = \frac{(3\times10^6)^2 \times (2.25\times10^{-4})}{4\times120\pi}\times 0.745 = 10^6 \text{(W)} = 1 \text{ (MW)}.$$

P.10-22 Let $A = \frac{1}{\eta_0 b}\sqrt{\frac{\pi f_c \mu_c}{\sigma_c}}$ and $x = \frac{f_c}{f}$ in Eq. (10-157).

We write $(\alpha_c)_{TE_{10}} = AF(x)$, where $F(x) = \frac{1 + \frac{2b}{a}x^2}{\sqrt{x(1-x^2)}}$.

For min. $(\alpha)_{TE_{10}}$, set $\frac{dF(x)}{dx} = 0$.

$$\longrightarrow x = \frac{f_c}{f} = \sqrt{\frac{1}{2}}\left[\left(1+\frac{a}{2b}\right) - \sqrt{\left(1+\frac{a}{2b}\right)^2 - \frac{2a}{4b}}\right]^{1/2}.$$

P.10-23 Field expressions for TM_{11} mode from Eqs. (10-132) and (10-136) through (10-137):

$$E_x^0(x,y) = -\frac{j\beta_{11}}{h^2}\left(\frac{\pi}{a}\right)E_0 \cos\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right),$$
$$E_y^0(x,y) = -\frac{j\beta_{11}}{h^2}\left(\frac{\pi}{b}\right)E_0 \sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right),$$
$$E_z^0(x,y) = E_0 \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right),$$
$$H_x^0(x,y) = \frac{j\omega\epsilon}{h^2}\left(\frac{\pi}{b}\right)E_0 \sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right),$$
$$H_y^0(x,y) = -\frac{j\omega\epsilon}{h^2}\left(\frac{\pi}{a}\right)E_0 \cos\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right).$$

Calculate α_c from Eq. (10-152): $\alpha_c = \frac{P_L(z)}{2P(z)}$.

$$P(z) = \frac{1}{2}\int_0^b\int_0^a \left[E_x^0 H_y^0 - E_y^0 H_x^0\right]dx\,dy = \frac{\omega\epsilon\beta E_0^2 ab}{8\left[\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2\right]}.$$

From problem P.10-13:

$$\bar{J}_s(y=0) = \bar{J}_s(y=b) = -\bar{a}_z \frac{j\omega\epsilon}{h^2}\left(\frac{\pi}{b}\right)E_0 \sin\left(\frac{\pi x}{a}\right)e^{-j\beta_{11}z},$$
$$\bar{J}_s(x=0) = \bar{J}_s(x=a) = -\bar{a}_z \frac{j\omega\epsilon}{h^2}\left(\frac{\pi}{a}\right)E_0 \sin\left(\frac{\pi x}{a}\right)e^{-j\beta_{11}z}.$$

$$P_L(z) = 2\left[P_L(z)\right]_{x=0} + 2\left[P_L(z)\right]_{y=0}.$$
$$\left[P_L(z)\right]_{x=0} = \frac{1}{2}\int_0^b |\bar{J}_s(x=0)|^2 R_s\,dy = \frac{(\omega\epsilon)^2 R_s}{4h^4}\left(\frac{\pi}{a}\right)^2 E_0^2 b,$$
$$\left[P_L(z)\right]_{y=0} = \frac{1}{2}\int_0^a |\bar{J}_s(y=0)|^2 R_s\,dx = \frac{(\omega\epsilon)^2 R_s}{4h^4}\left(\frac{\pi}{b}\right)^2 E_0^2 a.$$

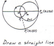
$$P_L(z) = \frac{(\omega\epsilon)^2 R_s E_0^2}{2\left[\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2\right]}\left[\left(\frac{\pi}{a}\right)^2 b + \left(\frac{\pi}{b}\right)^2 a\right].$$

$$\therefore\ (\alpha_c)_{TM_{11}} = \frac{2R_s\left(b/a^2 + a/b^2\right)}{\eta ab\sqrt{1-(f_c/f)^2}\left(1/a^2 + 1/b^2\right)}.$$

Min. $(\alpha_c)_{TM_{11}}$ occurs at $(f_c/f) = \sqrt{3}$.

P.10-24 $f = 10^{10}$ (Hz), $f_c = \frac{u}{2a} = \frac{3\times10^8}{2\times0.0229} = 6.55\times10^9$ (Hz)

$\lambda_g = \frac{\lambda}{\sqrt{1-(f_c/f)^2}} = 0.0397$ (m) $= 3.97$ (cm) for TE_{10} mode.



Min.$|E|$ at $\frac{6}{3.97} = 1.511\lambda_g$ from the load, which is represented by the point P_m at $S = 1.80$.

Use Smith admittance chart:

Draw circle centered at O through P_m, intersecting the $g=1$ circle at P_1. Read at P_1: $y_1 = 1 - j0.60$.

Draw a straight line from O through P_1, intersecting the perimeter at P_1'. Read 0.152 on the "wavelengths toward load" scale. $\therefore$ P_1' is at $(0.250+0.152)\lambda_g = 0.402\lambda_g$ from P_m, the position of a min.$|E|$. In other words, P_1', the desired location of the iris, should be at $(1.511-0.402)\lambda_g$, or 4.40 (cm), from the load.

From Eq.(10-159): Capacitive iris to supply $b = +0.60$,

$$0.60 = \frac{4b}{\lambda_g}\ln\left[\csc\left(\frac{\pi d}{2b}\right)\right] \longrightarrow d = 0.384 \text{ (cm)}.$$

P.10-26 TM modes: $E_z = E_z^0 e^{-\gamma z}$, $H_z = 0$.

$\nabla\times\bar{E} = -j\omega\mu\bar{H}$		$\nabla\times\bar{H} = j\omega\epsilon\bar{E}$	
$\frac{\partial E_z^0}{r\partial\phi} + \gamma E_\phi^0 = -j\omega\mu H_r^0$.	①	$\gamma H_\phi^0 = j\omega\epsilon E_r^0$.	④
$-\gamma E_r^0 - \frac{\partial E_z^0}{\partial r} = -j\omega\mu H_\phi^0$.	②	$-\gamma H_r^0 = j\omega\epsilon E_\phi^0$.	⑤
$\frac{1}{r}\left[\frac{\partial}{\partial r}(rE_\phi^0) - \frac{\partial E_r^0}{\partial\phi}\right] = 0$.	③	$\frac{1}{r}\left[\frac{\partial}{\partial r}(rH_\phi^0) - \frac{\partial H_r^0}{\partial\phi}\right] = j\omega\epsilon E_z^0$	⑥

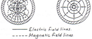
Eliminating H_ϕ^0 from ② and ④: $E_r^0 = -\frac{\gamma}{h^2}\frac{\partial E_z^0}{\partial r}$. ⑦

Eliminating H_r^0 from ① and ⑤: $E_\phi^0 = -\frac{\gamma}{h^2}\frac{\partial E_z^0}{r\partial\phi}$. ⑧

Combining ⑦ and ⑧: $(\bar{E}_T^0)_{TM} = \bar{a}_r E_r^0 + \bar{a}_\phi E_\phi^0 = -\frac{\gamma}{h^2}\left(\bar{a}_r\frac{\partial E_z^0}{\partial r} + \bar{a}_\phi\frac{\partial E_z^0}{r\partial\phi}\right)$

$$= -\frac{\gamma}{h^2}\nabla_T E_z^0.$$

P.10-27 a) TM_{11} b) TE_{01}



—— Electric field lines.
- - - - Magnetic field lines

c) Eq. (10-35): $f_c = \frac{h}{2\pi\sqrt{\mu\epsilon}} = \frac{1.5\times10^8}{\pi}h$

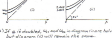
For TM_{11} mode, $(h)_{TM_{11}} = \frac{3.832}{a} \longrightarrow (f_c)_{TM_{11}} = \frac{1.83\times10^8}{a}$ (Hz)

For TE_{01} mode, $(h)_{TE_{01}} = \frac{3.832}{a} \longrightarrow (f_c)_{TE_{01}} = (f_c)_{TM_{11}}$.
(Degenerate mode

P.10-28 $\beta^2 = k^2 - h^2 = \omega^2\mu\epsilon - h^2$

For TE_{11} mode, $h = x'_{11}/a = 1.841/a$,
$\beta = \sqrt{(\omega/c)^2 - (1.841/a)^2}$, $\omega_{c1} = 1.841c/a$.

For TM_{01} mode, $h = x_{01}/a = 2.405/a$,
$\beta = \sqrt{(\omega/c)^2 - (2.405/a)^2}$, $\omega_{c2} = 2.405c/a$.



a) If a is doubled, ω_{c1} and ω_{c2} in diagram (i) are halved; but diagram (ii) will remain the same.

b) If the waveguide medium is changed from (ϵ_0, μ_0) to $(\epsilon_0\epsilon_r, \mu_0)$, both ω_{c1} and ω_{c2} are reduced by a factor $\sqrt{\epsilon_r}$ and the slope of the asymptotic line is changed from c to $u_p = c/\sqrt{\epsilon_r}$; diagram (ii) remains unchanged.

P.10-29 a) TM modes: $E_z^0 = C_n J_n(hr)\sin n\phi$.

Boundary conditions $E_z^0 = 0$ at both $\phi = 0$ and $\phi = \pi$ are satisfied when n is an integer.

There are no TM_{0p} modes.

b) TE modes: $H_z^0 = C_n J_n(hr)\cos n\phi$, where n is an integer.

TE_{0p} modes can exist.

c) For TM modes, b.c. at $r=a$ requires that $J_n(ha)=0$.

$\longrightarrow$ Eigenvalues $(h)_{TM_{np}} = x_{np}/a$, $n=1,2,3,\cdots$

For TE modes, b.c. at $r=a$ requires that $J_n'(ha)=0$.

$\longrightarrow$ Eigenvalues $(h)_{TE_{np}} = x'_{np}/a$, $n=0,1,2,\cdots$

P.10-30 From Eqs. (10-245) and (10-249):

Inside the slab: $\beta^2 = \omega^2\mu_d\epsilon_d - k_y^2 < \omega^2\mu_d\epsilon_d$.

Outside the slab: $\beta^2 = \omega^2\mu_0\epsilon_0 + \alpha^2 > \omega^2\mu_0\epsilon_0$.

$$\therefore \quad \omega\sqrt{\mu_0\epsilon_0} < \beta < \omega\sqrt{\mu_d\epsilon_d},$$

and

$$\frac{1}{\sqrt{\mu_0\epsilon_0}} > u_p = \frac{\omega}{\beta} > \frac{1}{\sqrt{\mu_d\epsilon_d}}.$$

P.10-31 From Eqs. (10-258) and (10-2 7):

$$\left(\frac{\alpha d}{2}\right)^2 + \left(\frac{k_y d}{2}\right)^2 = \left(\frac{k_0 d}{2}\right)^2\left(\frac{\mu_d\epsilon_d}{\mu_0\epsilon_0} - 1\right). \qquad ①$$

$$\frac{\epsilon_d}{\epsilon_0}\left(\frac{\alpha d}{2}\right) = \left(\frac{k_y d}{2}\right)\tan\left(\frac{k_y d}{2}\right). \qquad ②$$

Let $X = k_y d/2$, $Y = \alpha d/2$, $A = \epsilon_0/\epsilon_d$, and $R = \frac{k_0 d}{2}\sqrt{\frac{\mu_d\epsilon_d}{\mu_0\epsilon_0} - 1}$.

Eqs. ① and ② become
$$\begin{cases} X^2 + Y^2 = R^2, & ③ \\ Y = AX\tan X. & ④ \end{cases}$$

a) $f = 2\times10^8$ (Hz), $\lambda = c/f = 1.5$ (m).

$k_0 d/2 = \pi d/\lambda = 0.0209$, $A = \epsilon_0/\epsilon_d = 0.385$, $R = 0.0314$.

Graphical solution:

$X_0 = 0.0314$, $Y_0 = 3.038 \times 10^{-4}$

$\alpha = 2Y_0/d = 0.061$ (Np/m).

$k_y = 2X_0/d = 6.28$ (rad/m).

From Eq. (10-245): $\beta = \sqrt{\omega^2\mu_d\epsilon_d - k_y^2} = 4.19$ (rad/m).

b) $f = 5\times10^8$ (Hz), $\lambda = c/f = 0.60$ (m), $k_0 d/2 = 0.0524$.

$A = 0.308$, $R = 0.0785$,

$X_0 = 0.0785$, $Y_0 = 1.901\times10^{-3}$

We obtain $\alpha = 0.380$ (Np/m),

$\beta = 10.48$ (rad/m).

P.10-32 From Eq. (10-263):

$$\left(\frac{\alpha d}{2}\right) = -\frac{\epsilon_0}{\epsilon_d}\left(\frac{k_y d}{2}\right)\cot\left(\frac{k_y d}{2}\right). \qquad ②$$

Using the notations in problem P.10-31, we obtain two equations from ① in P.10-31 and ② above:

$$\begin{cases} X^2 + Y^2 = R^2, & ③ \\ Y = -AX\cot X. & ④ \end{cases}$$

a) $f = 2\times10^8$ (Hz), $\lambda = 1.5$ (m). $A = 0.308$, $R = 0.0314$.

b) $f = 5\times10^8$ (Hz), $\lambda = 0.60$ (m). $A = 0.308$, $R = 0.075$.

There are no intersections for curves representing Eqs. ③ and ④; hence even TM modes do not exist at the given frequencies.

P.10-33 Use Eqs. (10-28) and (10-25):

$$E_y^0 = -\frac{j\beta}{h^2}\frac{\partial E_z^0}{\partial y}, \qquad H_x^0 = \frac{j\omega\epsilon}{h^2}\frac{\partial E_z^0}{\partial y}.$$

$$\bar{E}(y,z;t) = \mathcal{R}e\left[\bar{E}^0(y)\,e^{j(\omega t-\beta z)}\right].$$

$$\bar{H}(y,z;t) = \mathcal{R}e\left[\bar{H}^0(y)\,e^{j(\omega t-\beta z)}\right].$$

$|y| \le d/2$:

$E_z^0(y) = E_0 \cos k_y y \longrightarrow E_z(y,z;t) = E_0 \cos k_y y \cos(\omega t - \beta z)$,

$E_y^0(y) = \frac{j\beta}{k_y} E_0 \sin k_y y \longrightarrow E_y(y,z;t) = -\frac{\beta}{k_y} E_0 \sin k_y y \sin(\omega t - \beta z)$,

$H_x^0(y) = -\frac{j\omega\epsilon_d}{k_y} E_0 \sin k_y y \longrightarrow H_x(y,z;t) = \frac{\omega\epsilon_d}{k_y} E_0 \sin k_y y \sin(\omega t - \beta z)$.

$y \ge d/2$:

$E_z^0(y) = E_0 \cos\left(\frac{k_y d}{2}\right) e^{-\alpha(y-\frac{d}{2})} \longrightarrow E_z(y,z;t) = E_0 \cos\left(\frac{k_y d}{2}\right) e^{-\alpha(y-\frac{d}{2})} \cos(\omega t - \beta z)$,

$E_y^0(y) = -\frac{j\beta}{\alpha} E_0 \cos\left(\frac{k_y d}{2}\right) e^{-\alpha(y-\frac{d}{2})} \longrightarrow E_y(y,z;t) = \frac{\beta}{\alpha} E_0 \cos\left(\frac{k_y d}{2}\right) e^{-\alpha(y-\frac{d}{2})} \sin(\omega t - \beta z)$,

$H_x^0(y) = \frac{j\omega\epsilon_0}{\alpha} E_0 \cos\left(\frac{k_y d}{2}\right) e^{-\alpha(y-\frac{d}{2})} \longrightarrow H_x(y,z;t) = -\frac{\omega\epsilon_0}{\alpha} E_0 \cos\left(\frac{k_y d}{2}\right) e^{-\alpha(y-\frac{d}{2})} \sin(\omega t - \beta z)$.

$y \le -d/2$:

$E_z^0(y) = E_0 \cos\left(\frac{k_y d}{2}\right) e^{\alpha(y+\frac{d}{2})} \longrightarrow E_z(y,z;t) = E_0 \cos\left(\frac{k_y d}{2}\right) e^{\alpha(y+\frac{d}{2})} \cos(\omega t - \beta z)$,

$E_y^0(y) = \frac{j\beta}{\alpha} E_0 \cos\left(\frac{k_y d}{2}\right) e^{\alpha(y+\frac{d}{2})} \longrightarrow E_y(y,z;t) = -\frac{\beta}{\alpha} E_0 \cos\left(\frac{k_y d}{2}\right) e^{\alpha(y+\frac{d}{2})} \sin(\omega t - \beta z)$,

$H_x^0(y) = -\frac{j\omega\epsilon_0}{\alpha} E_0 \cos\left(\frac{k_y d}{2}\right) e^{\alpha(y+\frac{d}{2})} \longrightarrow H_x(y,z;t) = \frac{\omega\epsilon_0}{\alpha} E_0 \cos\left(\frac{k_y d}{2}\right) e^{\alpha(y+\frac{d}{2})} \sin(\omega t - \beta z)$.

P.10-34 a) From Table 10-4 on p. 572 it is seen that $f_{c0} = 0$ for TE_1 mode, which is the dominant mode.

From Eq. (10-275):

$$\alpha = \frac{\mu_0}{\mu_d} k_y \tan\frac{k_y d}{2} \cong \frac{\mu_0 d}{2\mu_d} k_y^2, \text{ for } k_y d \ll 1.$$

Neglecting the α^2 term in Eq. (10-267):

$$\beta^2 - \omega^2\mu_0\epsilon_0 = \alpha^2 \cong 0 \longrightarrow \beta \cong \omega\sqrt{\mu_0\epsilon_0} = k_0.$$

From Eq. (10-265): $k_y^2 = \omega^2\mu_d\epsilon_d - \beta^2 = k_d^2 - k_0^2$.

$$\therefore \quad \alpha \cong \frac{\mu_0 d}{2\mu_d}(k_d^2 - k_0^2).$$

b) $d = 5\times10^{-3}$ (m), $\epsilon_d = 3\epsilon_0$, $\mu_d = \mu_0$, $f = 3\times10^8$ (Hz), $k_0 = 2\pi$.

$$\alpha = \frac{d}{2} k_0^2 (\epsilon_r - 1) = 0.197 \quad (\text{Np/m}).$$

$$e^{-\alpha(y-\frac{d}{2})} = 0.368, \quad \alpha\left(y - \frac{d}{2}\right) = 1.$$

$$\longrightarrow \left(y - \frac{d}{2}\right) = 5.066 \text{ (m)}.$$

P.10-35 Use Eqs. (10-50) and (10-51):

$$H_y^0 = -\frac{j\beta}{h^2}\frac{\partial H_z^0}{\partial y}, \qquad E_x^0 = -\frac{j\omega\mu}{h^2}\frac{\partial H_z^0}{\partial y}.$$

$$\bar{H}(y,z;t) = \mathcal{R}e\left[\bar{H}^0(y)\,e^{j(\omega t-\beta z)}\right]$$

$$\bar{E}(y,z;t) = \mathcal{R}e\left[\bar{E}^0(y)\,e^{j(\omega t-\beta z)}\right].$$

$|y| \le d/2$:

$$H_z^0(y) = H_0\cos k_y y \longrightarrow H_z(y,z;t) = H_0\cos k_y y\cos(\omega t-\beta z),$$

$$H_y^0(y) = \frac{j\beta}{k_y}H_0\sin k_y y \longrightarrow H_y(y,z;t) = -\frac{\beta}{k_y}H_0\sin k_y y\sin(\omega t-\beta z),$$

$$E_x^0(y) = \frac{j\omega\mu_d}{k_y}H_0\sin k_y y \longrightarrow E_x(y,z;t) = -\frac{\omega\mu_d}{k_y}H_0\sin k_y y\sin(\omega t-\beta z).$$

$y \ge d/2$:

$$H_z^0(y) = H_0\cos\left(\frac{k_y d}{2}\right)e^{-\alpha(y-\frac{d}{2})} \longrightarrow H_z(y,z;t) = H_0\cos\left(\frac{k_y d}{2}\right)e^{-\alpha(y-\frac{d}{2})}\cos(\omega t-\beta z),$$

$$H_y^0(y) = -\frac{j\beta}{\alpha}H_0\cos\left(\frac{k_y d}{2}\right)e^{-\alpha(y-\frac{d}{2})} \longrightarrow H_y(y,z;t) = \frac{\beta}{\alpha}H_0\cos\left(\frac{k_y d}{2}\right)e^{-\alpha(y-\frac{d}{2})}\sin(\omega t-\beta z),$$

$$E_x^0(y) = -\frac{j\omega\mu_0}{\alpha}H_0\cos\left(\frac{k_y d}{2}\right)e^{-\alpha(y-\frac{d}{2})} \longrightarrow E_x(y,z;t) = \frac{\omega\mu_0}{\alpha}H_0\cos\left(\frac{k_y d}{2}\right)e^{-\alpha(y-\frac{d}{2})}\sin(\omega t-\beta z).$$

$y \le -d/2$:

$$H_z^0(y) = H_0\cos\left(\frac{k_y d}{2}\right)e^{\alpha(y+\frac{d}{2})} \longrightarrow H_z(y,z;t) = H_0\cos\left(\frac{k_y d}{2}\right)e^{\alpha(y+\frac{d}{2})}\cos(\omega t-\beta z).$$

$$H_y^0(y) = \frac{j\beta}{\alpha}H_0\cos\left(\frac{k_y d}{2}\right)e^{\alpha(y+\frac{d}{2})} \longrightarrow H_y(y,z;t) = -\frac{\beta}{\alpha}H_0\cos\left(\frac{k_y d}{2}\right)e^{\alpha(y+\frac{d}{2})}\sin(\omega t-\beta z),$$

$$E_x^0(y) = \frac{j\omega\mu_0}{\alpha}H_0\cos\left(\frac{k_y d}{2}\right)e^{\alpha(y+\frac{d}{2})} \longrightarrow E_x(y,z;t) = -\frac{\omega\mu_0}{\alpha}H_0\cos\left(\frac{k_y d}{2}\right)e^{\alpha(y+\frac{d}{2})}\sin(\omega t-\beta z).$$

Setting $y = d/2$ in $E_x^0(y) = \frac{j\omega\mu_d}{k_y}H_0\sin k_y y$ and

in $E_x^0(y) = -\frac{j\omega\mu_0}{\alpha}H_0\cos\left(\frac{k_y d}{2}\right)e^{-\alpha(y-\frac{d}{2})}$,

and equating, we obtain

$$E_x^0\left(\frac{d}{2}\right) = \frac{j\omega\mu_d}{k_y}H_0\sin\left(\frac{k_y d}{2}\right) = -\frac{j\omega\mu_0}{\alpha}H_0\cos\left(\frac{k_y d}{2}\right).$$

$$\longrightarrow \quad \frac{\alpha}{k_y} = -\frac{\mu_0}{\mu_d}\cot\left(\frac{k_y d}{2}\right).$$

P.10-36 a) Odd TM and even TE modes are the propagating modes. Using $2d$ for d in the formulas in Table 10-3, p.578, we have

$$f_{cn} = \frac{n-1}{2d\sqrt{\mu_d\epsilon_d-\mu_0\epsilon_0}} \quad \text{for odd TM modes,}$$

$$f_{cn} = \frac{n-\frac{1}{2}}{2d\sqrt{\mu_d\epsilon_d-\mu_0\epsilon_0}} \quad \text{for even TE modes.}$$

b) <u>For odd TM modes</u> — From Eqs. (10-127 b and c):

$|y| \leq d/2$. $\quad E_y^0(y) = -\frac{j\beta}{k_y} E_0 \cos k_y y$

$\quad H_x^0(y) = \frac{j\omega\epsilon_d}{k_y} E_0 \cos k_y y.$

Surf. current density on conductor $\bar{J}_s = \bar{a}_n \times \bar{H}\big|_{y=0}$

$$\bar{J}_s = -\bar{a}_z H_x^0(0) = -\bar{a}_z \frac{j\omega\epsilon_d}{k_y} E_0.$$

Surf. charge density on conductor $\rho_s = \bar{a}_n \cdot \bar{D}\big|_{y=0}$

$$\rho_s = \epsilon_d E^0(0) = -\frac{j\beta\epsilon_d}{k_y} E_0.$$

<u>For even TE modes</u> — From problem P.10-27:

$|y| \leq d$. $\quad H_y^0(y) = \frac{j\beta}{k_y} H_0 \sin k_y y$

$\quad E_x^0(y) = \frac{j\omega\mu_d}{k_y} H_0 \sin k_y y.$

$\quad H_z^0(y) = H_0 \cos k_y y.$

$\therefore\ \bar{J}_s = \bar{a}_y \times [\bar{a}_y H_y^0(0) + \bar{a}_z H_z^0(0)] = \bar{a}_x H_0$

$\quad \rho_s = \bar{a}_y \cdot \epsilon_d \bar{E}(0) = 0.$

<u>P.10-37</u> For circularly symmetrical TE modes, $E_z^0 = 0$, $\frac{\partial}{\partial\phi} \to 0$.

a) For $r \leq a$: $E_{z1}^0 = 0$, $H_{z1}^0 = C_0 J_0(hr)$. $\quad h^2 = \omega^2\mu_1\epsilon_1 - \beta^2$.

$$\begin{cases} \bar{E}_T^0 = \frac{1}{h^2}(\bar{a}_z j\omega\mu_1 \times \bar{\nabla}_T H_{z1}^0) = \frac{j\omega\mu_1}{h^2} \bar{a}_z \times (\bar{a}_r \frac{\partial H_{z1}}{\partial r}) = \bar{a}_\phi \frac{j\omega\mu_1}{h} C_0 J_0'(hr). & (1) \\ \bar{H}_T^0 = -\frac{1}{h^2}(\gamma \bar{\nabla}_T H_{z1}^0) = -\frac{\gamma}{h^2}(\bar{a}_r \frac{\partial H_{z1}^0}{\partial r}) = -\bar{a}_r \frac{\gamma}{h} C_0 J_0'(hr). & (2) \end{cases}$$

From (1) & (2): $r \leq a$ $\begin{cases} E_{\phi 1}^0 = \frac{j\omega\mu_1}{h} C_0 J_0'(hr), \\ E_{r1}^0 = 0. \end{cases}$ $\quad H_{r1}^0 = -\frac{\gamma}{h} C_0 J_0'(hr),$ $\quad H_{\phi 1}^0 = 0,$ $\quad H_{z1}^0 = C_0 J_0(hr).$

Similarly, for $r \geq a$:

$\quad H_{z0}^0 = D_0 K_0(\xi r)$, where $\xi^2 = -(\omega^2\mu_0\epsilon_0 - \beta^2)$

$\quad \longrightarrow E_{\phi 0}^0 = -\frac{j\omega\mu_0}{\xi} D_0 K_0'(\xi r), \quad H_{r0}^0 = \frac{\gamma}{\xi} D_0 K'(\xi r).$

b) Boundary conditions at $r = a$ $\begin{cases} \text{for } H_z^0: & C_0 J_0(ha) = D_0 K_0(\xi a); & (3) \\ \text{for } E_\phi^0: & \frac{\mu_1}{h} C_0 J_0'(ha) = -\frac{\mu_0}{\xi} D_0 K_0'(\xi a). & (4) \end{cases}$

$\frac{(3)}{(4)} \longrightarrow$ Characteristic equation for symmetrical TE modes: $\quad \frac{J_0(ha)}{J_0'(ha)} = -\frac{\mu_1 \xi}{\mu_0 h} \frac{K_0(\xi a)}{K_0'(\xi a)}.$

P.10-38 From Eq. (10-301): $f_{mnp} = \frac{u}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2+\left(\frac{p}{d}\right)^2}$.

$$f_{mnp} = 1.5\times10^{10}F(m,n,p)\ ,\quad F(m,n,p)=\sqrt{\left(\frac{m}{8}\right)^2+\left(\frac{n}{6}\right)^2+\left(\frac{p}{5}\right)^2}.$$

Lowest-order modes and resonant frequencies:

Modes	$F(m,n,p)$	$(f_0)_{mnp}$ in (Hz)
TM_{110}	0.2083	3.125×10^9
TE_{101}	0.2358	3.538×10^9
TE_{011}	0.2603	3.905×10^9
TE_{111}, TM_{111}	0.2888	4.332×10^9
TM_{210}	0.3005	4.507×10^9
TE_{201}	0.3202	4.802×10^9
TM_{120}	0.3560	5.340×10^9
TE_{211}, TM_{211}	0.3609	5.414×10^9
TE_{021}	0.3887	5.831×10^9
TE_{121}, TM_{121}	0.4083	6.125×10^9

P.10-39 a) Since $d>a>b$, the lowest-order resonant mode is TE_{101} mode.

$$f_{101} = \frac{c}{2}\sqrt{\frac{1}{a^2}+\frac{1}{d^2}} = 4.802\times10^9\ (Hz).$$

b) From Eq. (10-322):

$$Q_{101} = \frac{\pi f_{101}\mu_0 abd(a^2+d^2)}{R_s[2b(a^3+d^3)+ad(a^2+d^2)]} \qquad \left(R_s=\sqrt{\frac{\pi f_{101}\mu_0}{\sigma}}\right)$$

$$= \frac{\sqrt{\pi f_{101}\mu_0\sigma}\ abd(a^2+d^2)}{2b(a^3+d^3)+ad(a^2+d^2)} = 6869.$$

From Eqs. (10-316) and (10-317):

$$W_e = \frac{1}{4}\epsilon_0\mu_0^2 a^3bd f_{101}^2 H_0^2 = 0.07728\times10^{-12}\ (J),$$

$$W_m = \frac{\mu_0}{16}abd\left(\frac{a^2}{d^2}+1\right)H_0^2 = 0.07728\times10^{-12}\ (J) = W_e.$$

P.10-40 a) $(f_{101})_{\epsilon_d} = \frac{u}{2}\sqrt{\frac{1}{a^2}+\frac{1}{d^2}} = \frac{1}{\sqrt{\epsilon_r}}(f_{101})_{\epsilon_0} = 3.037\times10^9$ (Hz).

b) $(Q_{101})_{\epsilon_d} = \frac{1}{(\epsilon_r)^{1/4}}(Q_{101})_{\epsilon_0} = 5462.$

c) $(W_e)_{\epsilon_d} = (W_e)_{\epsilon_0} = 0.07728\times10^{-12}$ (J) $= 0.07728$ (pJ)

$= (W_m)_{\epsilon_d}.$

P.10-41 a) Combining Eqs. (10-318) and (10-322):

$$Q_{101} = \frac{\pi\eta}{2R_s}\frac{b(a^2+d^2)^{3/2}}{[2b(a^3+d^3)+ad(a^2+d^2)]}.$$

⟶ Q_{101} has a symmetrical dependence on a and d. It will be maximum when $a=d$, which gives a max. volume-to-surface ratio.

b) When $a=d$, $Q_{101} = \frac{\pi\eta}{2R_s}\frac{1}{\sqrt{2}\,(1+b/2a)}.$

P.10-42 a) $Q_{101} = \frac{\sqrt{\pi f_{101}\mu_0\sigma}\;abd(a^2+d^2)}{2b(a^3+d^3)+ad(a^2+d^2)}.$

For $a=d=1.8b$, $f_{101} = \frac{1}{2\sqrt{\mu_0\epsilon_0}}\sqrt{\frac{1}{a^2}+\frac{1}{d^2}} = 1.179\times10^{8}\left(\frac{1}{b}\right).$

$Q_{101} = 10.22\sqrt{\sigma b}.$

b) For $Q'_{101} = 1.20\,Q_{101}$, $b' = 1.20^2 b = 1.44\,b.$

P.10-43 (I) From the field configurations in the cavity we see that the TM_{110} mode with respect to z is the same as the TE_{101} mode with respect to y. Thus, $(Q_{110})_{TM}$ can be obtained from $(Q_{101})_{TE}$ in Eq. (10-322) by changing b to d and d to b.

or, (II) Q for the TM_{110} mode can be derived from the field expressions in Eqs. (10-295, 298, & 299) by setting $m=n=1$, and using Eq. (10-315).

$W = 2W_m = \frac{\mu_0}{8}\left(\frac{\omega^2\epsilon_0^2}{h^4}\right)abd\,E_0^2$ at f_{110}.

$P_L = \oint \frac{1}{2}|\bar{J}_s|^2 R_s\,ds = \oint \frac{1}{2}|\bar{H}_t|^2 R_s\,ds.$

$$P_L = R_s\left\{\int_0^d\int_0^b |H_y(x=0)|^2\,dy\,dz + \int_0^d\int_0^a |H_x(y=0)|^2\,dx\,dz + \int_0^b\int_0^a \left[|H_x(z=0)|^2 + |H_y(z=0)|^2\right]dx\,dy\right\}$$

$$= \frac{R_s}{2}\left(\frac{\omega^2\epsilon^2}{h^4}\right)E_0^2\left\{\frac{1}{h^2}\left(\frac{\pi}{a}\right)^2 bd + \frac{1}{h^2}\left(\frac{\pi}{b}\right)^2 ad + \frac{1}{2}ab\right\},$$

$$h^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2.$$

$$Q_{110} = \frac{\omega_{110}W}{P_L} = \frac{\pi f_{110}\mu_0 abd(a^2+b^2)}{R_s\left[2d(a^3+b^3)+ab(a^2+b^2)\right]}, \quad R_s = \sqrt{\frac{\pi f_{110}\mu_c}{\sigma}}.$$

P.10-44 a) TM_{mnp} mode:

$$E_z^0 = C_m J_m\left(\frac{x_{mn}}{a}r\right)\cos m\phi \cos\left(\frac{p\pi}{d}z\right),$$

$$m = 0,1,2,\cdots;\ n = 1,2,3,\cdots;\ p = 0,1,2,\cdots$$

$$(f_{TM})_{mnp} = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{x_{mn}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2},$$

$$(\lambda_{TM})_{mnp} = \frac{2\pi}{\sqrt{\left(\frac{x_{mn}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2}}.$$

TE_{mnp} mode: $H_z^0 = C_m J_m\left(\frac{x'_{mn}}{a}r\right)\cos m\phi \sin\left(\frac{p\pi}{d}z\right),$

$$m = 0,1,2,\cdots;\ n = 1,2,3,\cdots;\ p = 1,2,3,\cdots$$

$$(f_{TE})_{mnp} = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{x'_{mn}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2},$$

$$(\lambda_{TE})_{mnp} = \frac{2\pi}{\sqrt{\left(\frac{x'_{mn}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2}}.$$

b) For $d=a$, the dominant mode is TM_{010}: $(f_{TM})_{010} = \frac{2.405}{2\pi a\sqrt{\mu\epsilon}} = \frac{0.38}{a\sqrt{\mu\epsilon}}$.

The first seven modes with the lowest resonant freqs:

Mode	TM_{010}	TE_{111}	TM_{110}	TM_{011}	TE_{211}	TM_{111}	TE_{011}
$f_r/(f_r)_{TM010}$	1.00	1.51	1.59	1.65	1.82	2.06	2.06

P.10-45 $C = \frac{\epsilon S}{d} = \frac{\epsilon\pi a^2}{d}, \quad L = \frac{\mu h}{2\pi}\ln\left(\frac{b}{a}\right).$

a) $f_r = \frac{1}{2\pi\sqrt{LC}} \cong \frac{1}{\pi a\sqrt{\mu\epsilon}\sqrt{\frac{2h}{d}\ln\left(\frac{b}{a}\right)}}.$

b) $\lambda_r = \frac{1}{f_r\sqrt{\mu\epsilon}} \cong \pi a\sqrt{\frac{2h}{d}\ln\left(\frac{b}{a}\right)}.$

Chapter 11

Antennas and Radiating Systems

P.11-1 Maxwell's equations for simple media:

$$\bar{\nabla} \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t}. \quad ①$$

$$\bar{\nabla} \times \bar{H} = \bar{J} + \epsilon \frac{\partial \bar{E}}{\partial t}. \quad ②$$

$$\bar{\nabla} \cdot \bar{E} = \frac{\rho}{\epsilon}. \quad ③$$

$$\bar{\nabla} \cdot \bar{H} = 0. \quad ④$$

a) $\bar{\nabla} \times ①$: $\bar{\nabla} \times \bar{\nabla} \times \bar{E} = -\mu \frac{\partial}{\partial t}(\bar{\nabla} \times \bar{H})$

$$= -\mu \frac{\partial \bar{J}}{\partial t} - \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2}. \quad ⑤$$

But $\bar{\nabla} \times \bar{\nabla} \times \bar{E} = \bar{\nabla}(\bar{\nabla} \cdot \bar{E}) - \bar{\nabla}^2 \bar{E}$

$$= \frac{1}{\epsilon} \bar{\nabla} \rho - \bar{\nabla}^2 \bar{E}. \quad ⑥$$

Combining ⑤ and ⑥, we obtain

$$\bar{\nabla}^2 \bar{E} - \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2} = \frac{1}{\epsilon} \bar{\nabla} \rho + \mu \frac{\partial \bar{J}}{\partial t}.$$

b) Similarly, we have $\bar{\nabla}^2 \bar{H} - \mu\epsilon \frac{\partial^2 \bar{H}}{\partial t^2} = -\bar{\nabla} \times \bar{J}$.

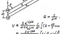
P.11-2 Eq. (11-2): $\bar{E} = -\bar{\nabla} V - j\omega \bar{A} = \bar{a}_R E_R + \bar{a}_\theta E_\theta + \bar{a}_\phi E_\phi$.

$$E_R = -\frac{\partial V}{\partial R} - j\omega A_R$$

$$E_\theta = -\frac{\partial V}{R\,\partial \theta} - j\omega A_\theta$$

$$E_\phi = -\frac{\partial V}{R \sin\theta\, \partial \phi} - j\omega A_\phi$$

The expressions of A_R, A_θ, and A_ϕ are given in Eqs. (11-14 a, b, and c).



$$V = \frac{Q}{4\pi\epsilon_0}\left[\frac{e^{-j\beta R_+}}{R_+} - \frac{e^{-j\beta R_-}}{R_-}\right];$$

$$R_+ \cong R - \frac{1}{2} d\ell \cos\theta,$$

$$R_- \cong R + \frac{1}{2} d\ell \cos\theta.$$

$$Q = \frac{I}{j\omega}, \quad (d\ell)^2 << R^2.$$

$$V = \frac{I e^{-j\beta R}}{4\pi\epsilon_0 j\omega} \frac{1}{R^2}\left[\left(R + \frac{d\ell}{2}\cos\theta\right) e^{j\beta(d\ell\cos\theta)/2} - \left(R - \frac{d\ell}{2}\cos\theta\right) e^{-j\beta(d\ell\cos\theta)/2}\right]$$

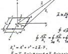
$$= \frac{I e^{-j\beta R}}{4\pi\epsilon_0 j\omega R^2}\left[2jR \sin\left(\frac{\beta\, d\ell \cos\theta}{2}\right) + 2\left(\frac{d\ell}{2}\cos\theta\right)\cos\left(\frac{\beta\, d\ell\cos\theta}{2}\right)\right]$$

$$V \cong \frac{I e^{-j\beta R}}{4\pi\epsilon_0 j\omega R^2}\left[2jR\left(\frac{\beta\, d\ell \cos\theta}{2}\right) + d\ell\cos\theta\right]$$

$$= \frac{I\, d\ell \cos\theta}{4\pi R^2}\eta_0\left(R + \frac{1}{j\beta}\right)e^{-j\beta R}.$$

Using A_R, A_θ, A_ϕ, and V in E_R, E_θ, and E_ϕ, we obtain the same results as given in Eqs. (11-16a,b,&c).

P.11-3



a) $$\bar{A} = \frac{\mu_0 I}{4\pi}\oint \frac{e^{-j\beta R_1}}{R_1}\,d\bar{\ell}'$$

$$= \frac{\mu_0 I}{4\pi}e^{-j\beta R}\oint \frac{e^{-j\beta(R_1-R)}}{R_1}\,d\bar{\ell}'.$$

$$\bar{A} \cong \frac{\mu_0 I}{4\pi}e^{-j\beta R}\left[(1+j\beta R)\oint\frac{d\bar{\ell}'}{R_1} - j\beta\oint d\bar{\ell}'\right].$$

$$\oint d\bar{\ell}' = 0.$$

$$\oint\frac{d\bar{\ell}'}{R_1} = \int_{AB}\frac{d\bar{\ell}'}{R_1} + \int_{CD}\frac{d\bar{\ell}'}{R_1} + \int_{BC}\frac{d\bar{\ell}'}{R_1} + \int_{DA}\frac{d\bar{\ell}'}{R_1}.$$

$$R_1^2 = R^2 + r^2 - 2\bar{R}\cdot\bar{r},$$

$$\bar{R} = \bar{a}_x R\sin\theta\cos\phi + \bar{a}_y R\sin\theta\sin\phi + \bar{a}_z R\cos\theta.$$

$$\bar{r} = \bar{a}_x x + \bar{a}_y \frac{L_y}{2}, \quad \bar{R}\cdot\bar{r} = Rx\sin\theta\cos\phi + R\frac{L_y}{2}\sin\theta\sin\phi,$$

$$\frac{1}{R_1} \cong \frac{1}{R}\left[1 + \frac{\bar{R}\cdot\bar{r}}{R^2}\right] = \frac{1}{R}\left(1 + \frac{x}{R}\sin\theta\cos\phi + \frac{L_y}{2R}\sin\theta\sin\phi\right).$$

$$\frac{\mu_0 I}{4\pi}e^{-j\beta R}(1+j\beta R)\int_{AB}\frac{d\bar{\ell}'}{R_1} = \bar{a}_x\frac{\mu_0 I}{4\pi}e^{-j\beta R}(1+j\beta R)\frac{1}{R}\int_{L_x/2}^{-L_x/2}\left(1+\frac{x}{R}\sin\theta\cos\phi + \frac{L_y}{2R}\sin\theta\sin\phi\right)dx$$

$$= \bar{a}_x\frac{\mu_0 I}{4\pi}e^{-j\beta R}(1+j\beta R)\frac{1}{R}\left(-L_x - \frac{L_x L_y}{2R}\sin\theta\sin\phi\right).$$

In the same manner, we have

$$\frac{\mu_0 I}{4\pi}e^{-j\beta R}(1+j\beta R)\int_{CD}\frac{d\bar{\ell}'}{R_1} = \bar{a}_x\frac{\mu_0 I}{4\pi}e^{-j\beta R}(1+j\beta R)\frac{1}{R}\left(L_x - \frac{L_x L_y}{2R}\sin\theta\sin\phi\right).$$

$$\therefore \frac{\mu_0 I}{4\pi}e^{-j\beta R}(1+j\beta R)\int_{AB+CD}\frac{d\bar{\ell}'}{R_1} = -\bar{a}_x\frac{\mu_0 I}{4\pi R^2}e^{-j\beta R}(1+j\beta R)L_x L_y\sin\theta\sin\phi,$$

and

$$\frac{\mu_0 I}{4\pi}e^{-j\beta R}(1+j\beta R)\int_{BC+DA}\frac{d\bar{\ell}'}{R_1} = \bar{a}_y\frac{\mu_0 I}{4\pi R^2}e^{-j\beta R}(1+j\beta R)L_x L_y\sin\theta\cos\phi.$$

Let $m = I L_x L_y = IS$.

$$\bar{A} = \frac{\mu_0 m}{4\pi R^2}e^{-j\beta R}(1+j\beta R)\sin\theta\,(-\bar{a}_x\sin\phi + \bar{a}_y\cos\phi)$$

$$= \bar{a}_\phi\frac{\mu_0 m}{4\pi R^2}e^{-j\beta R}(1+j\beta R)\sin\theta.$$

c) $\bar{H} = \frac{1}{\mu_0}\bar{\nabla}\times\bar{A} = \bar{a}_R H_R + \bar{a}_\theta H_\theta$. Expressions for H_R, H_θ,

b) $\bar{E} = \frac{1}{j\omega\epsilon_0}\bar{\nabla}\times\bar{H} = \bar{a}_\phi E_\phi$. and E_ϕ same as those given in Eqs. (11-26a, b, c).

In the far zone, $\beta R >> 1$, $1/(j\beta R)^2$ and $1/(j\beta R)^3$ terms can be neglected. We have the following instantaneous expressions; assuming $i(t) = I\cos\omega t$:

$$\bar{A}(R,\theta;t) = -\bar{a}_\phi \frac{\mu_0 m}{4\pi R}\beta\sin\theta\sin(\omega t-\beta R).$$

$$\bar{E}(R,\theta;t) = \bar{a}_\phi \frac{\omega\mu_0 m}{4\pi R}\beta\sin\theta\cos(\omega t-\beta R).$$

$$\bar{H}(R,\theta;t) = -\bar{a}_\theta \frac{m}{4\pi R}\beta^2\sin\theta\cos(\omega t-\beta R).$$

P.11-4 Far-zone electric field of elemental electric dipole:

$$E_\theta(R) = j\frac{I_0 L}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\eta_0\beta\sin\theta \longrightarrow E_\theta(R,t) = -\frac{I_0\eta_0\beta\sin\theta}{4\pi R}(L)\sin(\omega t-\beta R).$$

For the elemental magnetic dipole:

$$E_\phi(R) = \frac{\omega\mu_0 m}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\beta\sin\theta \longrightarrow E_\phi(R,t) = \frac{I_0\eta_0\beta\sin\theta}{4\pi R}\left(\frac{2\pi S}{\lambda}\right)\cos(\omega t-\beta R).$$

a) Thus,
$$\frac{E_\theta^2(R,t)}{\left(\frac{I_0\eta_0\beta\sin\theta}{4\pi R}\right)^2(L)^2} + \frac{E_\phi^2(R,t)}{\left(\frac{I_0\eta_0\beta\sin\theta}{4\pi R}\right)^2\left(\frac{2\pi S}{\lambda}\right)^2} = 1.$$

——— Elliptic polarization.

b) Circular polarization if $L = 2\pi S/\lambda$.

P.11-5 Equation of continuity: $\bar{\nabla}\cdot\bar{J} = -j\omega\rho \longrightarrow \rho_\ell = \frac{j}{\omega}\frac{dI(z)}{dz}$.

a) $I(z) = I_0\cos\beta z. \longrightarrow \rho_\ell = -j\frac{\beta}{\omega}I_0\sin\beta z = -j\frac{I_0}{c}\sin\beta z.$

b) $I(z) = I_0\left(1-\frac{2}{\lambda}|z|\right) \longrightarrow \rho_\ell = \begin{cases} -j\frac{2I_0}{\lambda\omega} & \text{for } z>0 \\ +j\frac{2I_0}{\lambda\omega} & \text{for } z<0. \end{cases}$

P.11-6 $\lambda = \frac{3\times10^8}{10^6} = 300\ (\text{m})$, $\frac{d\ell}{\lambda} = \frac{15}{300} = \frac{1}{20} << 1$ (Hertzian dipole).

a) Radiation resistance, $R_r = 80\pi^2\left(\frac{d\ell}{\lambda}\right)^2 = 1.97\ (\Omega)$.

b) Eq. (11-49): $R_s = \sqrt{\frac{\pi f\mu_0}{\sigma}} = \sqrt{\frac{\pi 10^6(4\pi\times10^{-7})}{5.80\times10^7}} = 2.61\times10^{-4}\ (\Omega)$.

Eq. (11-48): $R_\ell = R_s\left(\frac{d\ell}{2\pi a}\right) = 0.031\ (\Omega) \longrightarrow \eta_r = \frac{R_r}{R_r+R_\ell} = 98.5\%$.

c) Eq. (11-43): $P_r = \frac{I^2(d\ell)^2}{12\pi}\eta_0\beta^2$; Eq. (11-19b): $|E_\theta|_{max.} = \left(\frac{I\,d\ell}{4\pi}\right)^2\frac{\eta_0^2\beta^2}{R^2}$ $\Bigg\} \longrightarrow |E_\theta|_{max.} = \frac{1}{R}\sqrt{90P_r} = 19\ (\text{mV/m})$.

P.11-7 a) $E_\theta = j\frac{I_0\eta_0\beta\sin\theta}{4\pi R}e^{-j\beta R}\int_{-h}^{h}\left(1-\frac{|z|}{h}\right)e^{j\beta z\cos\theta}dz$

$$= j\frac{I_0\eta_0\beta\sin\theta}{2\pi R}e^{-j\beta R}\int_0^h\left(1-\frac{z}{h}\right)\cos(\beta z\cos\theta)dz$$

$$= \frac{j60I_0}{(\beta h)R}e^{-j\beta R}F(\theta)$$

$(R_1 \cong R - z\cos\theta)$

$$H_\phi = \frac{E_\theta}{\eta_0} = \frac{jI_0}{(\beta h)2\pi R}e^{-j\beta R}F(\theta).$$

$$F(\theta) = \frac{\sin\theta[1-\cos(\beta h\cos\theta)]}{\cos^2\theta}.$$

In case $\beta h \ll 1$, $\cos(\beta h\cos\theta) \cong 1 - \frac{1}{2!}(\beta h\cos\theta)^2$, and

$$F(\theta) \cong \frac{1}{2}(\beta h)^2\sin\theta.$$

$$\therefore\quad E_\theta = \frac{j60I_0}{R}e^{-j\beta R}\left(\frac{1}{2}\beta h\sin\theta\right) = \frac{j30\beta h}{R}I_0e^{-j\beta R}\sin\theta.$$

$$H_\phi = \frac{jI_0}{2\pi R}e^{-j\beta R}\left(\frac{1}{2}\beta h\sin\theta\right) = \frac{j\beta h}{4\pi R}I_0e^{-j\beta R}\sin\theta.$$

b) $P_r = \frac{1}{2}\int_0^{2\pi}\int_0^{\pi}E_\theta H_\phi^* R^2\sin\theta\,d\theta\,d\phi = \frac{I_0^2}{2}\left[80\pi^2\left(\frac{h}{\lambda}\right)^2\right],$

$R_r = P_r/\left(\frac{1}{2}I_0^2\right) = 20\pi^2\left(\frac{2h}{\lambda}\right)^2.$

c) $D = \frac{4\pi|E_{max}|^2}{\int_0^{2\pi}\int_0^{\pi}|E_\theta(\theta)|^2\sin\theta\,d\theta\,d\phi} = \frac{2}{\int_0^{\pi}\sin^3\theta\,d\theta} = 1.5 \longrightarrow 10\log_{10}D = 1.76\,(dB).$

P.11-8 $I(z) = I_0\left(1-\frac{z}{h}\right).$

a) $\ell_e = \int_0^h\left(1-\frac{z}{h}\right)dz = \frac{h}{2} = 20\,(m)$, over conducting earth.

b) From Eq. (11-19b): $|E_\theta|_{max} = \frac{I_0}{4\pi R}\eta_0\beta\times 2\ell_e = \frac{30I_0\beta h}{R} = 2.52$ (m

c) $P_r = \frac{1}{2}\int_0^{\pi/2}\frac{|E_\theta|^2}{\eta_0}2\pi R^2\sin\theta\,d\theta = \frac{2\pi}{3}\eta_0\left(\frac{I_0\beta h}{4\pi}\right)^2 = 1.14\,(kW).$

d) $R_r = 2P_r/I_0^2 = 0.227\,(\Omega).$

P.11-9 a) $R_r = 80\pi^2\left(\frac{\beta\pi b^2}{\lambda}\right)^2 = 20(\beta^2\pi b^2)^2 = 320\pi^6\left(\frac{b}{\lambda}\right)^4.$

b) $\eta_r = \frac{R_r}{R_r+R_\ell}$; $R_\ell = \frac{b}{a}R_s$, $R_s = \sqrt{\pi f\mu_0/\sigma}$, $\sigma = 5.80\times10^7\,(S/m).$

c) $R_r = 320\pi^6\left(\frac{0.5}{300}\right)^4 = 2.35\times10^{-6}\,(\Omega).$

From problem P.11-6, $R_s = 2.61\times10^{-3} \longrightarrow R_\ell = 0.0435$, $\eta_r = 0.0055\%$

d) For $N=10$, $R_r(\propto N^2) = 2.35\times10^{-4}\,(\Omega)$, $R_\ell(\propto N) = 0.435\,(\Omega)$

$\longrightarrow \eta_r = 0.055\%$ (increases approximately as N).

P.11-10 a) $R_r = 80\pi^2\left(\frac{\beta L_x L_y}{\lambda}\right)^2 = 20\,(\beta^2 L_x L_y)^2$.

b) $R_\ell = \frac{L_x + L_y}{\pi b} R_s$, $\quad \eta_r = \frac{R_r}{R_r + R_\ell}$.

c) For $f = 1$ (MHz), $L_x = L_y = 1$ (m), and $a = 0.003$ (m)

$\beta = 2\pi/300$ (rad/m), $R_s = 2.61\times10^{-4}$ (Ω) from P.11-6.

$R_r = 3.85\times10^{-6}$ (Ω), $\quad R_\ell = 0.055$ (Ω).

$\longrightarrow \eta_r = 0.007\%$

P.11-11 $P_r = \oint \bar{\mathcal{P}}_{av}\cdot d\bar{s} = \frac{1}{2}\mathcal{R}e\int_0^{2\pi}\int_0^{\pi} E_\theta H_\phi^* R^2 \sin\theta\, d\theta\, d\phi$

$$= \frac{(I\,d\ell)^2}{16\pi}\beta^4 R^2 \eta_0 \mathcal{R}e\left\{\left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2}\right]\left[-\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2} - \frac{1}{(j\beta R)^3}\right]\right\}\cdot\int_0^{\pi}\sin^3\theta\, d\theta$$

$= \frac{I^2}{2}\left[80\pi^2\left(\frac{d\ell}{\lambda}\right)^2\right]$, which is the same as Eq.(11-43).

P.11-12 $F(\theta) = \frac{\cos(\beta h\cos\theta) - \cos\beta h}{\sin\theta}$.

For $2h = 1.25\lambda$,

$$|F(\theta)| = \left|\frac{\cos(1.25\pi\cos\theta) - \cos(1.25\pi)}{\sin\theta}\right|$$

Width of main beam between the first nulls $= 2\times 53.13° = 106.26°$.

P.11-13 $I(z) = I_0\left(1 - \frac{|z|}{h}\right)$.

From Eq.(11-70): $\ell_e = \frac{\sin\theta}{I_0}\int_{-h}^{h} I(z)\, e^{j\beta z\cos\theta}\, dz$

$$= \frac{\sin\theta}{I_0}\, 2\int_0^h I_0\left(1 - \frac{z}{h}\right)\cos(\beta z\cos\theta)\, dz$$

$$= \frac{2\sin\theta}{h(\beta^2\cos^2\theta)}\left[1 - \cos(\beta h\cos\theta)\right].$$

Maximum ℓ_e occurs at $\theta = \frac{\pi}{2}$, where

$$\ell_e\left(\frac{\pi}{2}\right) = h = \frac{\lambda}{12}.$$

P.11-14 a) $V_{oc} = -E_\theta \ell_e = -\frac{\lambda E_\theta}{\pi}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]$.

b) $P_L = \frac{1}{2}\left|\frac{V_{oc}}{R_r+R_L}\right|^2 R_L = \frac{|V_{oc}|^2}{8R_r} = \frac{\lambda^2 E_\theta^2}{8\pi^2 R_r}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]^2$,

which has a maximum value $(\lambda^2 E_\theta^2/8\pi^2 R_r)$ at $\theta = \frac{\pi}{2}$.

c) For $\lambda = \frac{3\times10^8}{1.5\times10^6} = 200$ (m) and $E_\theta = 5\times10^{-2}$ (V/m):

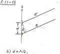
$\theta = \frac{\pi}{2} \longrightarrow \ell_e(\frac{\pi}{2}) = \frac{\lambda}{\pi} = 63.7$ (m), $V_{oc} = -3.15$ (V),

$P_L = \frac{3.15^2}{8\times73.1} = 0.017$ (W) $= 17$ (mW).

$\theta = \frac{\pi}{4} \longrightarrow \ell_e(\frac{\pi}{4}) = \frac{\lambda}{\pi}\left[\frac{\cos(\frac{\pi}{2}\cos45°)}{\sin45°}\right] = 40$ (m),

$V_{oc} = -2$ (V), $P_L = 6.8$ (mW).

P.11-15



$R' \cong R - d\cos\theta$

a) $E_\theta = \frac{jI(2h)}{4\pi}\eta_0\beta\sin\theta \cdot e^{-j\beta R} \cdot (1 + e^{j\beta d\cos\theta})$

$= \frac{j60Ih}{R}2\beta e^{-j\beta(R-\frac{d}{2}\cos\theta)}F(\theta)$,

where $F(\theta) = \sin\theta\cos(\frac{\beta d}{2}\cos\theta)$.

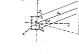
b) $d = \lambda/2$,

$|F(\theta)| = \left|\sin\theta\cos(\frac{\pi}{2}\cos\theta)\right|$.

c) $d = \lambda$,

$|F(\theta)| = \left|\sin\theta\cos(\pi\cos\theta)\right|$.

P.11-16



From Eq. (11-19b):

$$E_{\psi_1} = \frac{j I_0 d\ell\, \eta_0 \beta}{4\pi R_0} e^{-j\beta(R_0 - d\cos\theta)} \sin\psi,$$

$$E_{\psi_2} = -\frac{j I_0 d\ell\, \eta_0 \beta}{4\pi R_0} e^{-j\beta(R_0 + d\cos\theta)} \sin\psi,$$

where

$$\sin\psi = |\bar{a}_R \times \bar{a}_y|$$
$$= |(\bar{a}_x \sin\theta\cos\phi + \bar{a}_y \sin\theta\sin\phi + \bar{a}_z \cos\theta) \times \bar{a}_y|$$
$$= |\bar{a}_z \sin\theta\cos\phi - \bar{a}_x \cos\theta|$$
$$= \sqrt{1 - \sin^2\theta \sin^2\phi}.$$

$$E_\psi = E_{\psi_1} + E_{\psi_2} = j\frac{I_0 d\ell}{2\pi}\left(\frac{e^{-j\beta R_0}}{R_0}\right)\eta_0\beta \sin(\beta d\cos\theta)\sqrt{1-\sin^2\theta\sin^2\phi}.$$

Pattern function $F(\theta,\phi) = |\sin(\beta d\cos\theta)|\sqrt{1-\sin^2\theta\sin^2\phi}.$

a) In the xy-plane: $\theta = 90°$, $F_{xy}(90°,\phi) = 0.$

b) In the xz-plane: $\phi = 0°$, $E_\phi = 0$, $F_{xz}(\theta, 0°) = |\sin(\beta d\cos\theta)|.$

c) In the yz-plane: $\phi = 90°$, $E_\phi = 0$, $F_{yz}(\theta, 90°) = |\sin(\beta d\cos\theta)\cdot\cos\theta|.$

d) $d = \lambda/4$, $\beta d = \pi/2$:

$F_{xz}(\theta, 0°) = |\sin(\frac{\pi}{2}\cos\theta)|.$

$F_{yz}(\theta, 90°) = |\sin(\frac{\pi}{2}\cos\theta)\cos\theta|.$

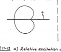
P.11-17 From Eq. (11-83): $|E| = \frac{2E_m}{R_0}\left|F(\theta,\phi)\cos\frac{\psi}{2}\right|$, where

$$\psi = \beta d \sin\theta\cos\phi + \xi .$$

In the H-plane of a dipole, $\theta = \pi/2$, $F(\frac{\pi}{2},\phi) = 1$.

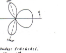
a) $d = \frac{\lambda}{4}$, $\xi = \frac{\pi}{2}$.

$|f(\phi)| = \left|\cos\left[\frac{\pi}{4}(1+\cos\phi)\right]\right|$.



b) $d = \frac{3\lambda}{4}$, $\xi = \frac{\pi}{2}$.

$|f(\phi)| = \left|\cos\left(\frac{3\pi}{4}\cos\phi + \frac{\pi}{4}\right)\right|$.



P.11-18 a) Relative excitation amplitudes: 1:4:6:4:1.

b) Array factor: $|A(\phi)| = \left|\cos\left(\frac{\pi}{2}\cos\phi\right)\right|^4$.

c) $\cos\left(\frac{\pi}{2}\cos\phi\right) = (\sqrt{2})^{-1/4}$

$\longrightarrow \phi = 74.86°$.

Half-power beamwidth

$= 2(90° - 74.86°)$

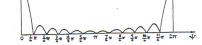
$= 30.28°$.

For uniform array, from Eq. (11-89):

$$\frac{1}{5}\left|\frac{\sin\left(\frac{5\pi}{2}\cos\phi\right)}{\sin\left(\frac{\pi}{2}\cos\phi\right)}\right| = \frac{1}{\sqrt{2}} \longrightarrow \phi = 79.61°$$

Half-power beamwidth for 5-element uniform array with $\lambda/2$ spacing $= 2(90° - 79.61°) = 20.78°$.

P.11-19 a) From Eq. (11-89) for $N=12$: $|A_N| = \frac{1}{12}\left|\frac{\sin 6\psi}{\sin(\psi/2)}\right|$.



b) <u>Broadside Operation.</u> $\psi = \beta d \cos\phi$.

$$|A(\psi)| = \frac{1}{N}\left|\frac{\sin(N\psi/2)}{\sin(\psi/2)}\right| \cong \left|\frac{\sin X}{X}\right| \quad \text{for } \psi \ll 1,$$

where $X = N\psi/2$.

At half-power points: $\left|\frac{\sin X}{X}\right| = \frac{1}{\sqrt{2}} \longrightarrow X = 1.391$

(for both broadside & endfire operations).

For broadside operation, the half-power beamwidth is

$$(2\Delta\phi)_{1/2} = 0.886\left(\frac{\lambda}{Nd}\right) \text{ (rad)}$$
$$= 50.75\left(\frac{\lambda}{Nd}\right) \text{ (deg)}.$$

For $N = 12$, $(2\Delta\phi)_{1/2} = 4.23\left(\frac{\lambda}{d}\right)$ (deg).

From Eq. (11-92): $(2\Delta\phi)_0 = 9.55\left(\frac{\lambda}{d}\right)$ (deg).

c) <u>Endfire Operation.</u> $\psi = \beta d(\cos\phi - 1)$.

$$(2\Delta\phi)_{1/2} = 1.882\sqrt{\frac{\lambda}{Nd}} \text{ (rad.)} = 107.8\sqrt{\frac{\lambda}{Nd}} \text{ (deg)}.$$

For $N = 12$, $(2\Delta\phi)_{1/2} = 31.13\sqrt{\frac{\lambda}{d}}$ (deg).

From Eq. (11-93): $(2\Delta\phi)_0 = 46.78\sqrt{\frac{\lambda}{d}}$ (deg).

<u>P.11-20</u> $|A(\psi)| = \frac{1}{N}\left|\frac{\sin(N\psi/2)}{\sin(\psi/2)}\right| \cong \left|\frac{\sin X}{X}\right|$, where $X = \frac{-N\psi}{2}$.

Assume broadside operation: $\psi = \beta d \cos\theta$.

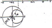
$$D = \frac{4\pi|A(\psi)_{max}|^2}{\int_0^{2\pi}\int_0^{\pi}\left|\frac{\sin X}{X}\right|^2 \sin\theta\, d\theta\, d\phi},$$

$$|A(\psi)_{max}| = 1.$$

$$\int_0^{\pi}\left|\frac{\sin X}{X}\right|^2 \sin\theta\, d\theta \cong \frac{4}{N\beta d}\int_0^{\infty}\left|\frac{\sin X}{X}\right|^2 dX = \frac{4}{N\beta d}\left(\frac{\pi}{2}\right) = \frac{\lambda}{Nd}.$$

$$\therefore D = \frac{2Nd}{\lambda} \cong \frac{2L}{\lambda}, \text{ where } L = \text{array length}.$$

<u>P.11-21</u> Construction follows the steps outlined on pp. 629–630.



P_0 is at $\xi = -\frac{\pi}{2}$.

Radius of circle is $\beta d = \frac{2\pi}{\lambda}\left(\frac{\lambda}{4}\right) = \frac{\pi}{2}$.

P.11-22 a) $A(\psi) = 1 + \zeta$, $\zeta = e^{j\psi}$. ψ_0 at π

b) $A(\psi) = (1+\zeta)^2$. A double-zero $\psi_{01} = \psi_{02} = \pi$.

c) $A(\psi) = \frac{\zeta^5 - 1}{\zeta - 1}$. Zeros at ψ_{0m} at $\frac{2\pi}{5}m$, $m = 1, 2, 3, 4$.

d) $A(\psi) = (1 + \zeta + \zeta^2)^2 = \left(\frac{\zeta^3 - 1}{\zeta - 1}\right)^2$

Double zeros $\begin{cases} \psi_{01} = \psi_{02} = +2\pi/3, \\ \psi_{03} = \psi_{04} = -2\pi/3. \end{cases}$

e) The zeros of an array polynomial specify the nulls in the array pattern as ϕ changes from 0 to π. $\zeta = 1$ ($\psi = 0$) gives the main beam. The regions between the nulls (except the main-beam region) are sidelobes. The double-zeros of the array in part (d) are more widely apart leading to a wider beamwidth and lower sidelobes (which are 3dB lower than those of a three-element uniform array).

P.11-23 From Eqs. (11-58) and (11-89):

$$|E_\theta| = \frac{j60 I_m N_1 N_2}{R} e^{-j\beta R} \left| \frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta} A_x(\psi_x) A_y(\psi_y) \right|$$

where $A_x(\psi_x) = \frac{1}{N_1} \frac{\sin(N_1\psi_x/2)}{\sin(\psi_x/2)}$, $\quad \psi_x = \frac{\beta d_1}{2} \sin\theta \cos\phi$;

$A_y(\psi_y) = \frac{1}{N_2} \frac{\sin(N_2\psi_y/2)}{\sin(\psi_y/2)}$, $\quad \psi_y = \frac{\beta d_2}{2} \sin\theta \cos\phi$.

$$|F(\theta,\phi)| = \frac{1}{N_1 N_2} \left| \left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right] \frac{\sin(\frac{N_1\psi_x}{2}) \sin(\frac{N_2\psi_y}{2})}{\sin(\frac{\psi_x}{2}) \sin(\frac{\psi_y}{2})} \right|.$$

P.11-24 From Eqs. (11-32) and (11-34): $G_D(\theta,\phi) = \frac{4\pi U(\theta,\phi)}{W_r} = \frac{4\pi R^2 \mathcal{P}_{av}}{W_r}$ ①

Using Eqs. (11-60) & (11-62) in ①: $G_D(\theta) = 1.64 \left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]^2$. ②

a) Substituting ② in Eq. (11-121): $A_e(\theta) = \frac{\lambda^2}{4\pi} G_D(\theta) = 0.13\lambda^2 \left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]^2$.

b) Max. value of $A_e(\theta)$ in a) is $A_e(\pi/2) = 0.13\lambda^2$.

For $f = 10^8$ (Hz), $\lambda = c/f = 3$ (m), $A_e(\pi/2) = 1.17$ (m²).

P.11-25 a) Eq. (11-127): $E_s = -\beta^2 b^3 \left(\frac{e^{-j\beta r}}{r}\right)\left(\frac{\epsilon_r - 1}{\epsilon_r + 2}\right) E_i \sin\theta.$

$$P_s = \oint_S \vec{\mathcal{P}}_{av} \cdot d\vec{s} = \int_0^{2\pi}\int_0^{\pi} \frac{E_s^2}{2\eta_0} r^2 \sin\theta \, d\theta \, d\phi$$

$$= \frac{8\pi}{3}\beta^4 b^6 \left(\frac{\epsilon_r - 1}{\epsilon_r + 2}\right)^2 \left(\frac{E_i^2}{2\eta_0}\right) \quad (W).$$

b) $\sigma_s = \frac{P_s}{\mathcal{P}_i} = \frac{8\pi}{3}\beta^4 b^6 \left(\frac{\epsilon_r - 1}{\epsilon_r + 2}\right)^2 = \frac{3}{2}\sigma_{bs}.$

P.11-26 From Eq. (11-132): $\frac{P_L}{P_t} = \left(\frac{\lambda}{4\pi r}\right)^2 G_{D1} G_{D2}.$

a) For half-wave dipoles: $G_{D1} = G_{D2} = 1.64.$

$f = 3\times10^{9}$ (Hz), $\lambda = c/f = 1$ (cm), $r = 1.5\times10^{3}$ (m).

$P_L = 7.57\times10^{-9} P_t = 7.57\times10^{-7}$ (W) $= 0.757$ (μW).

b) For Hertzian dipoles: $G_{D1} = G_{D2} = 1.50.$

$P_L = 0.633$ (μW).

P.11-27 From p.641: Earth radius = 6,380 (km),

Geosynchronous orbit radius = 42,300 (km).

Geosynchronous orbit

$\psi = \sin^{-1}\left(\frac{6{,}380}{42{,}300}\right) = \sin^{-1} 0.151 = 8.7°$

$\psi' = 90° - 8.7° = 81.3°$

a) Two satellites would cover only $2\times(2\psi') = 325.2° < 360°.$

Use three satellites in the equatorial plane: $3\times(2\psi') = 487.8° > 360°.$

$\psi' < 90°$ ⟶ cannot cover the polar regions.

b) Let P_t = Power transmitted by satellite antenna.

p_c = Power density within the cone $= \frac{G_D}{4\pi r^2} P_t.$

Area of cone cap on earth $= \int_0^{2\pi}\int_0^{\psi} r^2 \sin\theta \, d\theta \, d\phi$

$= 2\pi r^2 (1 - \cos\psi) \cong 2\pi r^2 (\psi^2/2) = \pi (r\psi)^2.$

$\therefore P_t = \pi (r\psi)^2 p_c$ ⟶ $\psi = \frac{1}{r}\sqrt{P_t/\pi p_c} = 2/\sqrt{G_D}.$

Main-lobe beamwidth $= 2\psi = 4/\sqrt{G_D}.$

P.11-28 a) From Eq. (11-132): $P_t = \frac{(4\pi r)^2}{G_t G_r \lambda_t^2} P_L$.

$\lambda_t = \frac{3\times10^8}{14\times10^9} = 2.14\times10^{-2}$ (m), $G_t = 10^{(55/10)} = 3.16\times10^5$;

$\lambda_r = \frac{3\times10^8}{12\times10^9} = 2.50\times10^{-2}$ (m), $G_r = 10^{(35/10)} = 3.16\times10^3$;

$r = 3.65\times10^7$ (m), $P_L = 8\times10^{-12}$ (W).

$\longrightarrow P_t = 2.7$ (W).

b) From Eq. (11-136): $P_t = \frac{4\pi}{\sigma_{bs}}\left(\frac{\lambda_t r^2}{A_e}\right)^2 P_L$.

$A_e = \frac{\lambda_t^2}{4\pi} G_t = 15.7$ (m²). $\longrightarrow P_t = 1.13$ (GW).

P.11-29 a) From Eq. (11-132): $P_L = G^2\left(\frac{\lambda}{4\pi r}\right)^2 P_t'$, ①

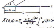
where, from Eq. (11-143), $P_t' = 4\left|\sin\left(\frac{2\pi h_1 h_2}{\lambda d}\right)\right|^2 P_t$
$= 0.0225\,P_t = 9$ (W).

Using ①: $P_L = 5.5\times10^{-10}$ (W) $= 0.55$ (nW).

b) Null field at $\frac{2\pi h_1 h_2}{\lambda d} = n\pi$, $n = 1, 2, 3, \ldots$

$\longrightarrow h_2 = 1.25n$ (km).

P.11-30



$I(z) = I_0 e^{-j\beta z}$

a) $\bar{A} = \bar{a}_z \frac{\mu}{4\pi}\int_0^L \frac{I_0 e^{-j\beta z} e^{-j\beta R'}}{R'}\,dz$.

In the far-zone, $R' \cong R - z\cos\theta$.

$\bar{A}(R,\theta) = \bar{a}_z \frac{\mu I_0 e^{-j\beta R}}{4\pi R}\int_0^L e^{-j\beta z(1-\cos\theta)}\,dz$

$= \bar{a}_z \frac{\mu I_0}{2\pi R\beta} e^{-j\beta R} e^{-j\beta\frac{L}{2}(1-\cos\theta)} F(\theta)$,

where $F(\theta) = \frac{\sin\left[\frac{\beta L}{2}(1-\cos\theta)\right]}{1-\cos\theta}$.

b) $A_R = A_z\cos\theta$, $A_\theta = -A_z\sin\theta$, $A_\phi = 0$.

$$\bar{H} = \frac{1}{\mu}\bar{\nabla}\times\bar{A} = \bar{a}_\phi \frac{1}{\mu R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right].$$

In the far-zone, $\frac{1}{R}\frac{\partial A_R}{\partial\theta} \propto \frac{1}{R^2} \longrightarrow \bar{H} = \bar{a}_\phi \frac{1}{\mu R}\frac{\partial}{\partial R}(RA_\theta)$

$$\bar{H}(R,\theta) = \bar{a}_\phi \frac{jI_0}{2\pi R} e^{-j\beta[R+L(1-\cos\theta)/2]} F(\theta)\sin\theta.$$

$$\bar{E}(R,\theta) = \bar{a}_\theta \eta_0 H_\phi(R,\theta).$$

c)

Radiation pattern for $L=\lambda/2$: Plot of $|F(\theta)\sin\theta|$.

P.11-31 From Eq. (11-74): $\bar{\ell}_e = \frac{\lambda}{\pi}(\bar{a}_x - \bar{a}_y j)$.

a) $\bar{E}_i = E_0(\bar{a}_x + \bar{a}_y jp)e^{jkz}$

From Eq. (11-76): $|V_{oc}| = |\bar{\ell}_e\cdot\bar{E}_i| = \frac{\lambda E_0}{\pi}(1+p) \longrightarrow 2\frac{\lambda E_0}{\pi}$, if $p=1$.

b) For $\bar{E}_i = E_0(\bar{a}_x - \bar{a}_y jp)e^{jkz}$,

$$|V_{oc}| = \frac{\lambda E_0}{\pi}(1-p) \longrightarrow 0, \text{ if } p=1.$$

c) For $\bar{E}_i = \bar{a}_x E_0 e^{jkz}$, $|V_{oc}| = \frac{\lambda E_0}{\pi}$.

P.11-32 Eq. (11-148): $\bar{E} = \frac{N\omega\mu_0 I}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)[\bar{a}_\theta js + \bar{a}_\phi \beta\pi b^2]\sin\theta.$

$$\bar{H} = \frac{1}{\eta_0}\bar{a}_R\times\bar{E} = \frac{N\beta I}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)[-\bar{a}_\theta\beta\pi b^2 + \bar{a}_\phi js]\sin\theta.$$

For circular polarization, $s = \beta\pi b^2$.

a) $U = R^2\mathscr{P}_{av} = R^2\bar{a}_R\cdot\bar{\mathscr{P}}_{av} = R^2\bar{a}_R\cdot\frac{1}{2}\mathcal{R}e(\bar{E}\times\bar{H}^*)$

$= \frac{\beta^2\eta_0}{16\pi^2}(NIs)^2\sin^2\theta.$

$P_r = \int_0^{2\pi}\int_0^{\pi} U\sin\theta\, d\theta\, d\phi = \frac{\beta^2\eta_0}{6\pi}(NIs)^2.$

$\therefore G_D = \frac{4\pi U}{P_r} = \frac{3}{2}\sin^2\theta;\quad D = G_D(\pi/2) = 1.5$

b) $R_r = 2P_r/I^2 = \eta_0(N\beta s)^2/3\pi = \frac{4\pi}{3}\eta_0\left(\frac{Ns}{\lambda}\right)^2 = 40(N\beta^2\pi b^2)^2.$

P.11-33 Assume $\bar{E}_a(x',y') = \bar{a}_x 1$.

$$F(\theta,\phi) = \int_{-a/2}^{a/2} e^{j\beta x'\sin\theta\cos\phi}dx' \int_{-b/2}^{b/2} e^{j\beta y'\sin\theta\cos\phi}dy'$$

$$= ab\left[\frac{\sin(\frac{\beta a}{2}\sin\theta\cos\phi)}{\frac{\beta a}{2}\sin\theta\cos\phi}\right]\left[\frac{\sin(\frac{\beta b}{2}\sin\theta\cos\phi)}{\frac{\beta b}{2}\sin\theta\cos\phi}\right].$$

$$\bar{E}_p(\theta,\phi) = \frac{jab}{\lambda R_0}e^{-j\beta R_0}\left[\frac{\sin(\frac{\pi a}{\lambda}\sin\theta\cos\phi)}{\frac{\pi a}{\lambda}\sin\theta\cos\phi}\right]\left[\frac{\sin(\frac{\pi b}{\lambda}\sin\theta\cos\phi)}{\frac{\pi b}{\lambda}\sin\theta\cos\phi}\right](\bar{a}_\theta\cos\phi - \bar{a}_\phi\sin\phi)$$

P.11-34 From Eq. (11-161):

$$F(\theta,\phi) = \iint_{aper.} E_a(x',y')e^{j\beta\sin\theta(x'\cos\phi + y'\sin\phi)}dx'dy'$$

a) In the xz-plane, $\phi = 0°$:

$$F_{xz}(\theta) = b\int_{-a/2}^{a/2} f(x')e^{j\beta x'\sin\theta}dx'$$

$$= 2b\int_0^{a/2}\left(1-\frac{2}{a}x'\right)\cos(\beta x'\sin\theta)dx'$$

$$= ab\,\frac{1-\cos(\frac{\beta a}{2}\sin\theta)}{(\frac{\beta a}{2}\sin\theta)^2}. \qquad \text{Let } \psi = \frac{\beta a}{2}\sin\theta = \frac{\pi a}{\lambda}\sin\theta$$

$$F_{xz}(\psi) = \frac{ab}{2}\left[\frac{\sin(\psi/2)}{(\psi/2)}\right]^2.$$

b) Set $\left[\frac{\sin(\psi/2)}{(\psi/2)}\right]^2 = \frac{1}{\sqrt{2}} \longrightarrow \frac{\psi}{2} = 1.005.$

Half-power beamwidth $(2\Delta\theta)_{1/2} = 2\sin^{-1}(0.640\frac{\lambda}{a})$.

For $\lambda/a \ll 1$, $(2\Delta\theta)_{1/2} \cong 1.280\frac{\lambda}{a}$ (rad)

$= 73.3\frac{\lambda}{a}$ (deg.).

c) Set $\frac{\psi}{2} = \pi \longrightarrow \theta_{n1} = \sin^{-1}(\frac{2\lambda}{a}) \cong \frac{2\lambda}{a}$ (rad)

$= 114.6\frac{\lambda}{a}$ (deg).

d) First sidelobes occur at $\psi/2 = 3\pi/2$,

where $F_{xz} = \frac{ab}{2}\left[\frac{1}{3\pi/2}\right]^2$.

$\therefore$ Level of first sidelobes, $L_1 = -20\log_{10}\left(\frac{1}{3\pi/2}\right)^2 = 26.9$ (dB).

Comparison of Results:

	Uniform Distr.	Triangular Distr.
Pattern function	$ab\left(\frac{\sin\psi}{\psi}\right)$	$\frac{ab}{2}\left(\frac{\sin\frac{\psi}{2}}{\frac{\psi}{2}}\right)^2$
Half-power beamwidth	$50\frac{\lambda}{a}$ (deg)	$73.3\frac{\lambda}{a}$ (deg)
Location of first null	$57.3\frac{\lambda}{a}$ (deg)	$114.6\frac{\lambda}{a}$ (deg)
First-side-lobe level	13.3 (dB)	26.9 (dB)

P.11-35 a) In the xz-plane, $\phi = 0°$:

$$F_{xz}(\theta) = 2b\int_0^{a/2}\cos\left(\frac{\pi x'}{a}\right)\cos(\beta x'\sin\theta)\,dx'$$

$$= \frac{2ab}{\pi}\left[\frac{(\pi/2)^2\cos\psi}{(\pi/2)^2-\psi^2}\right], \quad \psi = \frac{\beta a}{2}\sin\theta = \frac{\pi a}{\lambda}\sin\theta.$$

b) Set $\dfrac{(\frac{\pi}{2})^2\cos\psi}{(\frac{\pi}{2})^2-\psi^2} = \dfrac{1}{\sqrt{2}} \longrightarrow \psi = 1.868.$

Half-power beamwidth $(2\Delta\theta)_{1/2} = 2\sin^{-1}(0.594\frac{\lambda}{a})$.

For $\lambda/a \ll 1$, $(2\Delta\theta)_{1/2} \cong 1.188\frac{\lambda}{a}$ (rad)

$= 68.1\frac{\lambda}{a}$ (deg).

c) Set $\psi = \frac{3\pi}{2} \longrightarrow \theta_{01} = \sin^{-1}\left(\frac{3\lambda}{2a}\right) \cong \frac{3\lambda}{2a}$ (rad)

$= 85.9\frac{\lambda}{a}$ (deg).

d) At first sidelobe, $\psi \cong 2\pi$:

$$L_1 = -20\log_{10}\frac{(\frac{\pi}{2})^2}{(\frac{\pi}{2})^2-(2\pi)^2} = 20\log_{10}15 = 23.5 \text{ (dB)}.$$

	Uniform Distr.	Cosine Distr.
Pattern function	$ab\left(\frac{\sin\psi}{\psi}\right)^2$	$\frac{2ab}{\pi}\frac{\cos\psi}{1-(\frac{2}{\pi}\psi)^2}$
Half-power beamwidth	$50\frac{\lambda}{a}$ (deg)	$68.1\frac{\lambda}{a}$ (deg)
Location of first null	$57.3\frac{\lambda}{a}$ (deg)	$85.9\frac{\lambda}{a}$ (deg)
First-side-lobe level	13.3 (dB)	23.5 (dB)

The following corrections should be made to <u>Field and Wave Electromagnetics</u> by David K. Cheng. We apologize for any inconvienience that may result.

ERRATA SHEET

"Field and Wave Electromagnetics" by David K. Cheng (2nd. Ed.)

P. vii, 2nd paragraph, 7th line: filters ⟶ fibers.

P. 9, Eq. (1-9): Add (in free space) after the boxed equation—as in Eq.(1-8).

P. 29, Fig. 2-15: Two dashed lines for the bottom face of the volume are left out. The dot for ds_z should be put at the center of the bottom face. The dot for ds_x should be put at the center of the inner face.

P. 44, Eq. (2-96): $a_z \longrightarrow a_z$

P. 70, problem P.2-34: $3x^2y^2 \longrightarrow 3x^2y^3$.

P. 125, Fig. 3-30: Add c after comma.

P. 146, problem P.3-10: density ⟶ intensity.

P. 257, Eq. (6-106), in the numerator: $- \longrightarrow +$.

P. 297, problem 6-6, 1st. line: Add 'a radius b and 'after 'having'.

p. 297, problem 6-9: (6-32) ⟶ (6-33).

p. 300, Eq. (6-224): Insert '-' sign after the '=' sign.

P. 345, Table 7-5, 5th line: The letter K in Ku is not sharp.

P. 374, Eq. (8-66): $c \longrightarrow c_z$; Eq. (8-68): $n_z \longrightarrow n_z$; 5th line from bottom: $n_z \longrightarrow n_z$.

P. 408, line 5: Insert (transmission) after the word 'refraction'.

P. 422, problem P.8-27: Insert (V/m) before the word 'is'.

P. 423, problem P.8-29, b): $\lambda/2 \longrightarrow \lambda_z/2$.

P. 424, problem 8-34, line 2: Change 'the wave frequency to be' to 'a constant electron density and a wave frequency equal to'.

P. 426, line 2: $\theta_z \longrightarrow \theta_z$; problem P.8-42, line 3: reflection ⟶ refraction.

P. 476, Fig. 9-18(b): Color the segment between 0 and j_y. (See Fig.9-20, p.477.)

P. 481, 3rd. line from bottom: Fig.9-24(d), ⟶ Fig.9-25(d).

P. 517, problem P.9-38: 3rd. line: $R_0 \longrightarrow R$; R missing in Fig.9-46.

P. 537, Fig.10-6: Insert colored —— before 'Electric field lines' and ⊙ ⊗ before 'Magnetic field lines'.

P. 542, Eq. (10-95): [illegible] $\longrightarrow$ [illegible] (2 places); Eq. (10-96): [illegible] $\longrightarrow$ [illegible].

P. 598, problem P.10-41, 1st line: Change "An air-filled' to 'A';
2nd line: Delete 'made of copper'.

P. 599, problem P.10-44(b): Change 'cutoff' to 'resonant'.

P. 625, paragraph 3, line 3: $\binom{N}{n} \longrightarrow \binom{N-1}{n}$.

P. 649, 1st. table, line 2: 0.451λ ⟶ 0.510λ.

P. 685, line 4: $r \geq b \longrightarrow r \geq c$.

P. 687, problem P.8-27 b): $\mathcal{P}_{av}$. (more space.)

P. 688, problem P.9-15 b): 0.114π ⟶ 0.432π.

P. 690, problem P.11-9: 32.0 ⟶ 320.

Back endpaper, left: $\oint_s \longrightarrow \oint_s$ (See Eq.2-115, p. 50 and Eq.2-143, p. 59.)
Larger circles.

Figures in which the registers of color overlays are off:

P.2, Fig. 1-1.(Colored line should be centered on the circle.)

P.16, Fig. 2-6.

P.46, Fig.25(b). (q should be at the center.)

P.68, Fig.2-34.

P.76, Fig. 3-1. (P_2 should be on the curve.)

P.83, Fig. 3-5. Point +q should be colored; the arrow should be moved up to touch +q; the point at P should be at P.) See Fig.3-14 on p.95.

P.179, Fig.4-16 (2nd vertical colored line should coincide with the V(0,y) axis. See Fig.4-20 on p.186.

P.261, Fig.6-18: The arrows should pass through the centers of the circles.

P.297, Fig.6-37; P.480, Fig.9-24; P.550, Fig.10-11; P.556, Fig.10-14.